Toward Real-Time Magnetospheric Mapping Based on Multi-Probe Space Magnetometer Data

Nikolai A. Tsyganenko
Raytheon STX Corporation, NASA Goddard Space Flight Center, Greenbelt, Maryland

Abstract. Flexible and realistic mathematical models of the Earth’s magnetospheric field, combined with fast fitting algorithms, can serve as a tool for converting the flow of simultaneous data from a “swarm” of flying space magnetometers into a dynamically evolving three-dimensional picture of the geomagnetic field lines. In this paper several important aspects of the project are discussed, such as the reliability of the reconstruction of the magnetospheric configurations, the role of the irregular fluctuations in the data, spatial coverage of the magnetosphere by the multi-probes, and an estimate of the number of the spacecraft needed to achieve a reasonably accurate monitoring of the global magnetosphere in real time. The reconstruction of global magnetospheric configurations was simulated using artificial “data” generated by combining a realistic model of the magnetosphere with added random noise. Even with a high noise level, comparable in magnitude to the external regular field, the reconstructed field line configurations were found quite close to the original ones, provided the multi-probe swarm had a sufficient coverage of the tail lobes. In another set of runs, reconstruction of disturbed field configurations was made, simulating conditions during the substorm growth and expansion phases. Overall, the real-time monitoring of the global magnetosphere based on multi-probe data is a feasible task, provided spacecraft orbits are thoughtfully planned towards that end.

1. Introduction

Global modeling and monitoring of the geomagnetic field structure has a unique place in Sun-Earth connection studies, since that field underlies all processes in the near-Earth space environment. It links the interplanetary medium with the upper atmosphere and ionosphere, guides energetic charged particles, channels low-frequency electromagnetic waves, confines the radiation belts, directs electric currents, controls the auroral plasma, and stores huge amounts of energy, intermittently dissipated in the course of magnetic substorms. Magnetic fields determine key properties of the geospace plasma, in particular, its anisotropy, which fact makes it possible to compare observations made in different regions of space by mapping them along magnetic field lines. This is why understanding the properties of the geospace plasma requires knowing the structure of the geomagnetic field and its dynamics and relation to the state of the solar wind. In this regard, the role of the realistic data-derived models has been compared to that of maps in the exploration of a new country [Stern and Tsyganenko, 1992].

A large constellation of satellites covering the global magnetosphere should make it possible, in principle, to monitor and visualize the actual field configuration in real time. It should provide simultaneous vector measurements of the magnetic field at many locations, say, once a minute. Combining such data with flexible mathematical model representations and with fast-fitting algorithms would make possible the dynamic reconstruction of magnetospheric currents and the production of a sequence of real-time three-dimensional maps approximating the varying global magnetic field.

With concurrent data on the state of the solar wind, it should be possible to trace and understand the complex relationship between the state of the interplanetary medium and the dynamics of the magnetosphere/ionosphere system, which is the central problem of magnetospheric physics.

From the viewpoint of practical (e.g., space weather) applications, the real-time monitoring of the configuration of \( \mathbf{B} \) in the near-equatorial nightside magnetosphere might allow us to predict developing substorms 30-60 min in advance, by monitoring the stretching of field lines in the near tail, which is a typical precursor of the substorm expansion.

2. Basics of the Approach and Feasibility Factors

The idea of a synoptic tracking of the global geomagnetic configuration is not new: almost three decades ago Roederer [1969] suggested monitoring the position of the subsolar magnetopause and the strength of near-Earth tail current, using simultaneous data from two geosynchronous spacecraft. Foreseeing promising developments of that technique in the future, Roederer wrote: “... Hopefully, one day we may have a reliable magnetospheric model whose time variations are established by means of standard patrols of the magnetosphere.”

In a sense, such monitoring is analogous to the way real-time weather maps are obtained. An essential difference, however, is that while the mapping of atmospheric circulation is monitored on the basis of a dense two-dimensional network of ground observatories and detailed remote sensing from space, the magnetospheric “patrol service” will have to rely on the data from a flying three-dimensional swarm of space magnetometers, whose spatial coverage is not only less complete but continuously changes in time and may become significantly nonuniform. Hence, a careful planning of the space probe orbits is needed; it will also be necessary to continuously estimate the uncertainties of the reconstructed model field line patterns, which are quite sensitive to inaccuracies in approximating the major field sources.

The feasibility of real-time data-based mapping depends on several factors.

1. From the viewpoint of the model, it should meet the following requirements:

   (i) The modeling algorithms should be relatively simple and computationally fast. The existing version of the global model [Tsyganenko, 1996] can be fitted to ~50,000 data records within 20-40 minutes on a 200 MHz computer. In the multi-probe monitoring, the number of simultaneous data at any time will not exceed 200-400 (an upper estimate of the number of the satellites), which brings the computation time down to tens of seconds. Based on the recent progress in the modeling methods [Tsyganenko, 1998] and assuming a continuing advance in computer performance, one can expect that the fitting time for a multi-point data sample will drop to a few seconds or even less in the near future. Hence, the project seems computationally feasible.

   (ii) The models should be flexible, i.e., be able to reproduce a wide variety of possible configurations of \( \mathbf{B} \). This is achieved by building the model from several “modules”, each representing the contribution
from one source of the magnetospheric field. The modules contain adjustable parameters which determine the amplitude and geometrical characteristics of each field source, e.g., thickness of the tail current sheet. Models should correctly accommodate the effects of the variable tilt of the Earth’s dipole. A powerful method has been recently developed, representing those effects by divergence-free deformations of the underlying magnetic field model [Tsyganenko, 1998]. The technique is very simple mathematically, it does not violate the shielding conditions, and the tilt variations it produces in the magnetopause shape turn out to resemble the ones actually taking place. Due to the expected sparsity of the multi-probe data and a relatively high level of noise, the modules should be mutually “quasi-orthogonal”. In other words, it should be impossible to come close to representing the variations due to one module by adjusting the others. For example, the adopted method for flexible representation of the tail field is to combine several modes with quite different electric current profiles.

(iii) The models should be sufficiently “global”, in the sense that they must provide a correct description of the field in the regions not covered by data (at least, in the current-free regions like tail lobes). In particular, the models should be properly shielded within a realistic solar-wind-dependent magnetopause. This requirement is met in recent models.

2. From the viewpoint of the fitting criterion, one has to keep in mind that the ultimate goal is to accurately map the magnetic field configuration. In other words, the modeled object are the field lines, and hence the crucial quantity is the direction of the magnetic field \( \mathbf{b} = \mathbf{B}/B \). In an imaginary ideal case of a smooth static field, approximated by an infinitely flexible model, a best-fit solution in terms of \( \mathbf{B} \) should also provide a perfect match to the observed field in terms of \( \mathbf{b} \). However, in reality we have to deal with large variations of \( \mathbf{B} \) in the modeling region, from very small values in the distant tail to quite strong magnetic field in the vicinity of Earth. Also, the data distribution can be quite nonuniform, and the distant field itself exhibits irregular fluctuations, whose magnitude can be comparable to that of the regular component. For these reasons, the results of the fitting, based on the directional merit function [Tsyganenko, 1995]

\[
\sigma = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (b_{\text{mod}}(\mathbf{r}_i) - b_{\text{meas}}(\mathbf{r}_i))^2}
\]

can be noticeably different from those obtained using a standard fitting of the full field vectors. The directional merit function (1) yields much more robust field configurations in the most interesting magnetospheric regions where \( \mathbf{B} \) is relatively weak, i.e., in the tail plasma sheet and near the polar cusps.

3. The numerical fitting procedure should be fast and stable; in particular, it should not divert the search into unphysical regions in the parameter space. There exists a rich arsenal of algorithms; one of the best is the Newton-Letam-Marquardt (NLM) method (see [Tsyganenko, 1995] for a brief description and more references.) As a rule, the NLM procedure converges extremely quickly and also allows the evaluation of the correlations between the parameters and the errors of their estimates.

4. From the viewpoint of data coverage, an important question is the critical number of the multi-probes and their distribution within the magnetosphere, required for a reliable reconstruction of the field structure. To get a rough initial estimate, we conducted a numerical experiment, in which the observed magnetic field was generated by a fully shielded magnetosphere model, based on that of Tsyganenko, [1996], and an artificial noise was added to the regular field, to simulate transient fluctuations. The fluctuations can be quite significant: as suggested by data-based modeling, typical values of the residual angular rms deviation given by (1) are on the order of \( \sim 0.3 \) [Tsyganenko, 1995]. Part of that “noise” is due to the substorm related variations, which could not be properly reproduced by the model. Other sources include the flapping of the plasma sheet, instabilities of the magnetopause, FTE’s, solitary tailward traveling pulses, etc. The next section describes the simulations and their results.

3. Recovery of the Global Field From Noisy Data: a Simulation Study

In this study, three possible configurations of the probing constellations were simulated. The first one included 84 probing spacecraft, distributed within the radial distance of 50 \( R_E \) (Figure 1). In this configuration, the probes covered a relatively wide range of \( Z \), so that both the plasma sheet and tail lobes were properly represented. The second configuration (Figure 2) differed from the first one only in that it covered mostly the near-equatorial region, while the tail lobes were underrepresented. Finally, a configuration was tried out containing the same number of the probes, but distributed within much smaller region around Earth (Figure 3). That option was intended to simulate a mission monitoring the near magnetosphere, so that more details could be resolved on the dynamics of the inner magnetosphere during substorms.

4. The Magnetic Field Model.

The magnetic field model used for generating the “data” for the simulation included contributions from the ring current, the cross-tail current sheet, and the magnetopause field which completely enclosed all field lines inside the boundary. Although the model used here [Tsyganenko, 1996] also covered contributions from the Region 1 and 2 Birkeland currents, they were not included in this multi-probe simulation, since the other components already provide a good approximation of the large-scale field.

In the first two cases the tail field module contained three “submodules”, in each of which the cross-tail current density decreased with distance at a different rate. The current in the first submodule had the most rapid change, peaking at \( X \approx -7 R_E \) and falling down to half of its maximal strength at \( X \approx -11 R_E \) and decreasing to half its peak value at \( X \approx -5 R_E \) and \( X \approx -40 R_E \). The second submodule had an intermediate scale, peaking at \( X \approx -11 R_E \) and decreasing to half its peak value at \( X \approx -5 R_E \) and \( X \approx -40 R_E \). The third submodule complemented the first two and provided the asymptotic magnetic field in the deep-tail: its current density was zero earthward of \( X \approx -10 R_E \), and it slowly increased tailward to an asymptotic level, corresponding to \( \sim 10nT \) lobe field at \( X \approx -80 R_E \). In all three cases the current sheet half-thickness at midnight meridian was chosen equal to \( 2 R_E \). The profiles of the electric current density for the three tail submodules are given in curves 1–3 of Figure 4.

In the third set of the runs, an additional tail submodule was introduced for modeling the pre-substorm enhancement of the near-Earth equatorial current. The current sheet half-thickness for this source was set at \( 1 R_E \), and the current was localized within a relatively narrow range of distances, so that it peaked at \( X \approx -7 R_E \) and fell down to \( \sim 20\% \) of its peak value at \( X \approx -5 R_E \) and \( X \approx -10 R_E \). The profile of that substorm current is shown by curve 4 (dashed) in Figure 4.

In order to simulate the magnetic field collapse after the onset of the substorm expansion, a model substorm current wedge (SCW) was
The chaotic fluctuations in actual data were simulated by a random noise field added to the regular model field. More specifically, at each of the spacecraft locations (numbered by the index $j=1,2,...,M$), contributions from the tail and ring current were modified by adding to each of three field components (numbered below by $i=1,2,3$) the term

$$A_n B_i R_{ij} \exp \left[ -\frac{(X - X_0)^2}{\Delta X^2} - \frac{Z^2}{\Delta Z^2} \right] .$$

(2)

$B_i$ is the original field component at the given location, and $A_{nj}$ is the normalized noise amplitude ($0 \leq A_{nj} \leq 1$), so that $A_{nj} = 0$ corresponds to a perfectly noiseless model field, while $A_{nj} = 1$ provides the fluctuation of the same magnitude as the regular field. The factor $R_{ij}$ is a random number between -1 and 1, generated individually for every pair of the indices $i$ and $j$ in the ranges $1 \leq i \leq 3$ and $1 \leq j \leq M$. The parameters $X_0$, $\Delta X$, and $\Delta Z$ control the spatial distribution of the noise amplitude; they were specified separately for different sources, as follows. For the shortest-scale tail mode, we chose $X_0 = -10 R_E$, $\Delta X = -20 R_E$, and $\Delta Z = 8 R_E$, assuming that (i) the average position of the maximum of the noise amplitude matches that of the electric current, (ii) the spatial spreading of the noise source in the $X$ direction is approximately the same as for the magnetic field strength, and (iii) the fluctuation amplitude decreases away from the equatorial plane on a scale length, comparable with the maximum half-thickness of the plasma sheet. Likewise, for the intermediate-scale tail term those factors were chosen as $X_0 = -20 R_E$, $\Delta X = 40 R_E$ and $\Delta Z = 8 R_E$, and for the asymptotic tail term $X_0 = -50 R_E$, $\Delta X = 40 R_E$, and $\Delta Z = 8 R_E$.

A similar modification was adopted for the ring current module. Here the additional noise term was approximated as

$$A_n R_{ij} \exp \left[ -\frac{R^2}{\Delta R^2} \right] ,$$

(3)

with $\Delta R = 10 R_E$. 

---

**Figure 1.** Positions of the probes in the $X$-$Z$ plane projection, used in the first set of the geomagnetic field reconstruction runs. The coverage in $Y$ direction spans the interval $-15 R_E \leq Y \leq 15 R_E$.

**Figure 2.** Same as in Figure 1, but for the near-equatorial multi-probe configuration.

**Figure 3.** Same as in Figure 1, but for the near-earth multiprobe configuration.

**Figure 4.** Profiles of the cross-tail electric current density, corresponding to four terms in the tail field model. 1 - the short-scale term, 2 - the intermediate-scale term, 3 - asymptotic distant-tail term, 4 - substorm related term for the modeling of the growth phase effects.
5. Results of the Simulation Runs

In all cases considered, the amplitudes of the tail and ring current terms used for generating the sample noisy “data” sets were chosen close to the ones derived from real data for average magnetospheric conditions (e.g., [Tsyganenko, 1995]). Using those values, three sets were created: (i) a “noiseless” set with \( A_p = 0 \), (ii) a moderately noisy set with \( A_p = 0.5 \), and a strongly noisy one with \( A_p = 1 \). These sets were then used as inputs for deriving new parameters by means of an iterative NLM code. The following subsections describe the results for the three configurations of multi-probes.

5.1. The “Wide” Multi-probe Configuration

The entire “swarm” in this case consisted of two parts: a near-Earth subset and the tailward one. The near-earth part was generated by placing the probes at 3 initially equidistant values of the radius \( R \) within the interval \( 4 \leq R \leq 8 \), 3 values of the latitude \( \Lambda \) (\( -40^\circ \leq \Lambda \leq 40^\circ \)), and 4 values of the longitude \( \Phi \) (\( 0^\circ \leq \Phi \leq 40^\circ \)), so that the number of the near-Earth probes was \( N_p = 3 \times 3 \times 4 = 36 \). The initial equidistant values of the coordinates were then modified by introducing moderate random shifts, simulating nonuniformity of the real probe distribution in space.

The tailward part was specified in a similar way, but in Cartesian GSM coordinates, so that the probes filled the interior of a rectangular box \(-40 \leq X \leq 10, -15 \leq Y \leq 15, -10 \leq Z \leq 10\). The total number of probes in the tailward part was thus 40, corresponding to 4 equidistant values of \( X \), 4 values of \( Y \), and 3 values of \( Z \). These positions were also moderately randomized in all three dimensions.

The effect of the noise amplitude upon the accuracy of the reconstructed field line configuration can be visualized by plotting the obtained best-fit values of the components of the model field direction vector \( \mathbf{b} = \mathbf{B}/B \) at the probe positions against their values in the original input “data”. In the case of zero noise amplitude, the best-fit values of the model parameters should coincide with those used for generating the “data” sample, and hence the scatter plots should shrink to straight lines; that limiting case was used as a test for the least squares fitting algorithm.

Figure 5 shows the scatter plots for three components of \( \mathbf{b} \) (from left to right) and for three values of the noise amplitude factor \( A_p \) in (2)–(3) (from top to bottom): \( A_p = 0 \) (noiseless “data”), \( A_p = 0.5 \) (moderate noise), and \( A_p = 1 \) (strong noise). As it should be, in the noiseless case the reconstructed field is identical to the initial model one. Adding moderate noise resulted in a significant scatter around the diagonal straight line. A further increase of the noise amplitude to \( A_p = 1 \) makes the scatter stronger, but a large fraction of the points is still concentrated relatively close to the diagonal line.

Figure 6 shows the configurations of the reconstructed field for the same three values of the noise amplitude. In spite of a significant difference in the shape of the nightside field lines, even in the case with \( A_p = 1 \) (right panel) the degree of tailward stretch is approximately the same as in the original field (left panel). We conclude that the adopted number of the probes can be deemed marginally sufficient in this case.

5.2. The “Near-equatorial” Multi-probe Configuration

Figure 7 displays the results of the field line reconstruction using the “near-equatorial” distribution of the probes, in which the spread of the spacecraft along \( Z \) axis is much smaller. The lack of data from the lobe region, where the magnetic field is stronger and more stable, results in a dramatic deterioration of the reconstruction accuracy, as manifested by a gross difference between the original and reconstructed

![Figure 5](image-url)  
Figure 5. Scatter plots of three components of the magnetic field direction vector, \( \mathbf{b} = \mathbf{B}/B \) at the locations of the 84 probes in Figure 1. The reconstructed model values and those from the noisy “data” set are plotted along the horizontal and vertical axes, respectively. The noise amplitude increases from top to bottom, as indicated on the panels.
field line pattern. Therefore, in planning the orbital coverage it is important to ensure a sufficient scatter of probes in the north-south direction to sample the tail lobes.

5.3. The “Near-magnetosphere” Configuration and Reconstruction of Substorm Effects

The placement of the probes in this case was defined in essentially the same way as in the “wide” configuration, but the tailward extension of the swarm in this case ended at \( X = -15R_\oplus \), as shown in Figure 3. As noted in Section 4, the tail field expansions here included an additional term, representing the growth phase stretching of the nightside field, while substorm expansion effects were modeled by adding the field of the SCW.

Figure 8 (a growth-phase configuration) and Figure 9 (a collapsed field, with the inner tail current sheet off and the SCW term added) show the original (left panels) and reconstructed (right panels) configurations. In both cases, the strong noise \( A_n = 1 \) was added to the smooth “data”: in spite of that, the original field configurations are recovered fairly accurately, as one can verify by comparing the shapes and the degree of stretching of individual field lines. We again conclude that, with the assumed number and spatial configuration of the probes, the reconstruction of the instantaneous global field is a feasible task.

6. Conclusions

We modeled a derivation of the global magnetospheric magnetic configurations, based on data from an orbiting network of space magnetometers. In the present approach, the input data were simulated by using the components of the magnetic field direction vector \( \mathbf{b} \), returned by a realistic model of the geomagnetic field at 84 locations within the magnetosphere. The “data” were modified by adding a nonuniform distribution of random noise with variable amplitude. Reconstruction of the model parameters was made from that “data” using a nonlinear least squares NLM algorithm.

The procedure was applied to three possible configurations of the “swarm” of sampling probes: (1) a “wide” configuration, spanning relatively large intervals of all three Cartesian GSM coordinates, (2) a “near-equatorial” configuration, in which the probes were concentrated near the plasma sheet where the noise amplitude was greatest, and (3) a “near-magnetosphere” configuration, limited to much closer distances on the nightside.

All the reconstruction runs were made for three values of the noise amplitude: \( A_n = 0 \) (no noise and, hence, an exact reproduction of the original field), \( A_n = 0.5 \) (a moderate noise level), and \( A_n = 1 \) (a strong noise). It was found that

1. The assumed number of the multi-probes (84) can be considered sufficient for resolving the global magnetospheric field structure.

---

**Figure 6.** Original (left panel) and reconstructed (center and right panels) field lines, obtained in the “wide” configuration of the probes. In spite of the difference in the individual line shapes, the reconstructed field patterns are close to the original one even for the maximal noise level.

**Figure 7.** Original (left) and reconstructed (right) field lines, for the “near-equatorial” probe configuration in Figure 2. The “data” set was created assuming the noise level \( A_n = 1 \). Note a strong difference in the tail field line structure.
with a reasonable accuracy, provided the spatial distribution of the probes is sufficiently uniform in \(X, Y\) and spread-out in the \(Z\) direction, so that the tail lobes are properly represented. A satisfactory reconstruction can be achieved even with the noise amplitude in the plasma sheet having the same amplitude as the regular field in the adjacent tail lobe region.

(2) Based on the same number of the multi-probes, but concentrated within a smaller near-Earth region, a reconstruction of the substorm related reconfiguration of the field was attempted. It was found that both the field stretching in the near tail during the growth phase, and its collapse due to the development of the substorm current wedge could be tracked with a fairly good accuracy.

(3) In the described simulations we assumed the magnetopause size and shape to be known from simultaneous monitoring of the solar wind pressure and IMF. Availability of the solar wind and IMF data is of crucial importance for this project: in case such data are missing, it could still be possible to derive the field structure (albeit with a lesser accuracy); however, scientific value of such monitoring would be questionable, since one would be unable to establish causal relationship between the observed dynamics of the magnetosphere and solar wind conditions.

(4) The described study is the first attempt to test the tools currently at our disposal and crudely evaluate the potential value of multi-probe data for data-based modeling. Much work still remains to be done, in order to realistically determine the feasibility of such use of multi-probes. In this regard, note that the assumed coverage of the magnetosphere (e.g., Figure 1) can be maintained only for a relatively short period: because of the Earth’s orbital motion, the probes will exit the magnetospheric tail in a few months. Hence, a continuous coverage over a prolonged period can be ensured only if a much larger number of the probes is planned to be launched.
Acknowledgments. The author thanks David Stern for his careful reading and many useful comments on the manuscript, T. Han, and R. J. N. Phillips for doing the math in the formalism section.

References

N.A. Tsyganenko, Raytheon STX Corporation, Code 690.2, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA. (e-mail: tsyganenko@lepvx3.gsfc.nasa.gov)