Genetic Stochastic Algorithm Application in Beam Dynamics Optimization Problem

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Abstract The article discusses the application of the genetic global search algorithm to the problem of beam dynamics optimization. The algorithm uses normal distribution to form new generations and provides covariance matrix adaptation during random search. The method is easy to use because does not require calculation of the covariance matrix. The algorithm is applied to global extremum search of the functional characterizing beam dynamics quality in linear accelerator. The extremal problem under study has a large number of variables; the objective function is multi-extreme. Therefore, the use of the stochastic method is preferred way to achieve the goal. The algorithm quickly converges and can be successfully used in solving multidimensional optimization problems, including its combination with directed methods. The optimization results are presented and discussed.

1 Genetic Stochastic Algorithm with Covariance Matrix Evolution

Genetic algorithms realize an iterative approach; each iteration deals with a generation of points (individuals). General scheme of stochastic methods of global

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optimization includes initial generation design and the way of transition to next generation. A goal is to provide the convergence of generation sequence to global extremum point. The various genetic algorithms are developed and widely used, among them covariance matrix adaptation evolution strategy (CMA-ES) [4, 14] and simulated annealing method [7], both providing convergence of generation sequence to global extremum point with probability 1.

This paper deals with genetic algorithm [5] belonging to the set of methods allowing covariance matrix evolution in the course of random search. The next generation is modeled using normal distribution of random test points. The special method of new generation modeling permits not to calculate covariance matrix.

Genetic algorithm with covariance matrix evolution

Consider the problem of global minimum search of the function $F(X)$ in the domain D of n-dimensional Euclidean space E^n:

$$\min_{X\in D} F(X).$$

Let l be the number of gereration, ε be the prescribed accuracy.

A. Initial generation: $l = 0$.

1. Modeling M random points $X_i,\ i = \overline{1,M}$ using uniform distribution in the domain D.
2. $F_{min}^{(0)} = \min_{i=1,\dots,M} F(X_i),\ \ X_{min}^{(0)} = arg\min_{i=1,\dots,M} F(X_i)$.

B. Transition to next generation.

1. Selection of m “the best” points $Y_1,\dots,Y_m$ among the poits $X_1,\dots,X_M$.
2. Introduction of new points

$$X_j = \frac{1}{m}\sum_{i=1}^{m} \eta_j^{(i)} \left(Y_i - X_{min}^{(l)}\right) + X_{min}^{(l)},\ j = \overline{1,M},$$

where $\eta_j^{(i)},\ i = \overline{1,m},\ j = \overline{1,M}$ are independent standard normal random variables.
$l := l+1$.
3. $F_{min}^{(l)} = \min_{i=1,\dots,M} F(X_i),\ \ X_{min}^{(l)} = arg\min_{i=1,\dots,M} F(X_i)$.
4. If $\frac{F_{min}^{(l-1)} - F_{min}^{(l)}}{F_{min}^{(l-1)}} < \varepsilon,$ then go to **Exit**, else goto **B**.

Exit.

The convergence of this algorithm for unimodal function is proved in [5]. Main features of the method are as follows:

- Since random variables $\eta_j^{(i)}$ are included in the expressions for vectors X_j, the covariance matrix varies from generation to generation and allows one to concentrate the sample in the region of the scattering ellipsoid with center $X_{min}^{(l)}$. Thus, test points (individuals) more often appear in the vicinity of the best population found at the previous generation.

- When finding random normally distributed sampling points, it is not necessary to calculate the covariance matrix and use it in the simulation. This is especially important when the search space dimension is large.

2 Beam Dynamics Model and Optimization Problem

Beam dynamics optimization presents a class of specific complex problems, and the approaches to their solving are varied. Directed methods are widely used if mathematical optimization model is smooth [9, 10, 11]. These methods may be successfully combined with global search algorithms; widely known example of stochastic one is Particle Swarm Optimization [8]; some applications are given in [1, 12, 16]. If there are several different optimization goals, a multiobjective approach is effective [3, 7, 13]. A number of researchers use genetic algorithms of global search [2, 3, 6]. However, the genetic algorithm described above with adaptation of the covariance matrix was not applied to the problem of beam dynamics optimization prior to our research [15].

Let us investigate longitudinal dynamics of relativistic beam in linear waveguide accelerator. Beam evolution is considered to be a complex of synchronous particle motion and the motion of particles of a beam [16]. The synchronous phase is supposed to change along the structure. This approach opens up additional opportunities for optimization of beam evolution [9, 11, 12, 16]. Beam dynamics equations without considering Coulomb forces are as follows:

$$\begin{aligned}
&\frac{d\xi_s}{d\tau} = \frac{(\beta\gamma)_s}{\sqrt{1+(\beta\gamma)_s^2}}, \quad \frac{d(\beta\gamma)_s}{d\tau} = -\alpha(\xi_s, u_1)\sin(\varphi_s(\xi_s, u_2)),\\
&\frac{d\xi}{d\tau} = \frac{(\beta\gamma)}{\sqrt{1+(\beta\gamma)^2}}, \quad \frac{d(\beta\gamma)}{d\tau} = -\alpha(\xi, u_1)\sin(\hat{\varphi} + \varphi_s(\xi, u_2)),\\
&\frac{d\hat{\varphi}}{d\tau} = 2\pi\left(\frac{\beta\gamma\sqrt{1+(\beta\gamma)_s^2}}{(\beta\gamma)_s\sqrt{1+(\beta\gamma)^2}} - 1\right)
\end{aligned} \tag{1}$$

with initial conditions

$$\begin{aligned}
&\xi_s(0) = \xi_{s0},\ (\beta\gamma)_s(0) = (\beta\gamma)_{s0},\ \xi(0) = \xi_0,\ (\beta\gamma)(0) = (\beta\gamma)_0,\\
&\hat{\varphi}(0) = \frac{2\pi\xi_0\sqrt{1+(\beta\gamma)_{s0}^2}}{(\beta\gamma)_{s0}}.
\end{aligned} \tag{2}$$

Here τ is reduced time; ξ, $\beta\gamma$, β, γ are reduced values of particle coordinate, impulse, velocity and energy correspondingly; index s marks the characteristics of a synchronous particle; the functions α and φ_s are accordingly the dimensionless amplitude of accelerating wave and synchronous phase; u_1 and u_2 are the vectors of control parameters; $\hat{\varphi}$ is particle phase deviation from synchronous phase.

The independent variable is introduced to be time analogue for convenient account of Coulomb field in future research.

Hamilton equations describing longitudinal oscillations of particles near the synchronous one [11] with dynamic variables presenting the differences between asynchronous and synchronous phases ($\psi = \varphi - \varphi_s$) and reduced energies ($p_\psi = \gamma - \gamma_s$) are as follows:

$$\frac{p_\psi}{d\xi} = -\frac{\partial H(\xi,\psi,p_\psi)}{\partial \psi}, \quad \frac{d\psi}{d\xi} = \frac{\partial H(\xi,\psi,p_\psi)}{\partial p_\psi},$$

where

$$H(\xi,\psi,p_\psi) = \pi(\gamma_s^2-1)^{-3/2}p_\psi^2 + V(\xi,\psi),$$
$$V(\xi,\psi) = -\alpha(\xi,u_1)\left(\cos\left(\psi+\varphi_s(\xi,u_2)\right)+\psi\sin\left(\varphi_s(\xi,u_2)\right)\right).$$

Under the assumption of adiabatic variation of the functions $\alpha(\xi,u_1)$, $\varphi_s(\xi,u_2)$, $(\gamma_s^2-1)^{-3/2}$ along the structure we have an equation for the separatrix that restricts the region of particle capture into acceleration mode [11]:

$$p_\psi = \pm\sqrt{(1/\pi)(\gamma_s^2-1)^{3/2}}\sqrt{V(\xi,-\pi-2\varphi_s(\xi,u_2))-V(\xi,\psi)}. \qquad (3)$$

Consider the problem of beam dynamics optimization by control parameters $u = (u_1,u_2)$ to provide high quality of bunching and accelerating of particle beam. Let us present the optimization objectives and corresponding quality criteria taking into account the experience [9, 11, 12].

1. The first objective is to provide the synchronous particle output reduced energy in the required interval $[\gamma_1,\gamma_2]$. The corresponding quality criterion is

$$K_1(u) = \begin{cases} (\gamma_s(L)-\gamma_1)^2, & \gamma_s(L) < \gamma_1, \\ 0, & \gamma_s(L) \in [\gamma_1,\gamma_2], \\ (\gamma_s(L)-\gamma_2)^2, & \gamma_s(L) > \gamma_2, \end{cases}$$

where L is device exit reduced coordinate.

2. The goal of minimizing beam energy spread at accelerator exit may be achieved by minimizing the criterion

$$K_2(u) = |N_a|^{-1}\sum_{n\in N_a}\left(\gamma_n(L)-\gamma_s(L)\right)^2,$$

where N_a is a set of numbers of model particles captured in acceleration mode, $|N_a|$ is the total number of captured particles, n is model particle number.

3. For output phase spread minimizing at device exit the following criterion is introduced:

$$K_3(u) = |N_a|^{-1}\sum_{n\in N_a}\left(\hat{\varphi}_n(L)-\overline{\varphi}(L)\right)^2,$$

where $\overline{\varphi}$ is the average deviation of particle phase from synchronous one.

4. The value of criterion $K_4(u)$ is a total penalty imposed on particles that have left their bunch or are outside the separatrix (3) at any cross-section of the structure.

5. It is advisable to minimize the defocusing factor influence at the stage of longitudinal motion optimization [9]. To realize this idea we introduce the functional $K_5(u)$ imposing a penalty when the value of defocusing factor exceeds the specified limit.

6. To provide monotonous bunching we impose the requirement of negative rate $dG(\xi)/d\xi$ of variation along the structure of the mean square spread of particle phases [9]. To satisfy the stated requirement, we minimize the criterion $K_6(u)$ presenting the accumulated penalty introduced for each point where $dG(\xi)/d\xi > 0$.

The resulting beam quality criterion is as follows:

$$K(u) = \sum_{j=1}^{6} b_j K_j(u), \tag{4}$$

where b_j, $j = \overline{1,6}$ are the weight constants. So beam dynamics optimization problem is formulated as a problem of criterion (4) minimization by control u. Note that this is a problem of joint optimization of program motion (of synchronous particle) and the ensemble of beam particle motions [9, 11, 12].

3 Numerical Results

Numerical simulation and optimization of longitudinal beam dynamics are performed for electron waveguide accelerator with accelerating wave length $10\,cm$, structure length $80\,cm$, channel radius $0.04\,m$ and average beam current $0.25A$. Due to the low injection energy $(80keV)$ initial energy spread influence is not taken into account. The effect of Coulomb repulsion can also be neglected.

The functions $\alpha(\xi)$ and $\varphi_s(\xi)$ are modeled by trigonometric polynomials, the components of the vectors u_1 and u_2 are the values of the derivatives of polynomials at grid points and the values $\alpha(0)$, $\varphi_s(0)$. This allows one to obtain smooth functions $\alpha(\xi)$ and $\varphi_s(\xi)$. The functions used before optimization are presented in Fig. 1 (dotted lines).

Beam dynamics optimization problem is reduced to criterion (4) minimization by control parameters (the components of u_1 and u_2). This extremal problem is treated using genetic stochastic algorithm with covariance matrix evolution. To apply the algorithm presented above assume X be the vector of control parameters $(dim(X) = 84)$, $F(X) = K(u)$. So, the search is carried out in multidimensional domain $D \in E^{84}$. Numerical optimization experience shows that the objective function has many closely spaced extrema. Therefore, to implement the algorithm, a sufficiently large number M of random vectors is necessary. The parameters of method are chosen as follows: $M = 1000$, $m = 50$.

The optimization performed provided significant decrease of objective functional value and appropriate beam dynamics improving. It took only 6 generations to achieve the required accuracy ($\varepsilon = 0.01$) of finding the extremum (see Table 1).

Table 1 Criterion decrease during optimization

Generation	1	2	3	4	5	6
K(u)	15.463	6.303	4.940	4.742	4.678	4.674

Regarding beam characteristics at device output, the optimization allowed to reduce the phase spread from 1.63 to 0.98 (radian), to reduce the relative energy spread from 0.36 to 0.31 and to increase the number of particles within the separatrix from 94% to 98%. Capture coefficient remained constant during optimization and is equal 96%. Synchronous particle reduced energy changed from 11.93 to 11.37 and after optimization belongs the demanded interval (11.3; 11.7). So the optimization provided beam quality improvement.

The functions $\alpha(\xi)$ and $\varphi_s(\xi)$ obtained after optimization are shown in Fig. 1 (solid lines). The plots presenting beam dynamics (for particles in acceleration mode) before optimization (left) and after optimization (right) are given in Fig. 2–4.

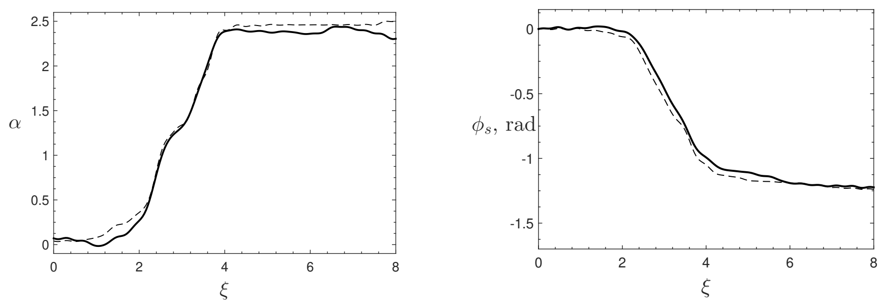

Fig. 1 The functions $\alpha(\xi)$ and $\varphi_s(\xi)$ before optimization (dashed line) and after optimization (solid line)

It should be noted that the genetic algorithm used is simple to implement, efficient and enables high performance. Comparative experiments have shown that the time spent for one iteration processing is two times less than the corresponding time for Particle Swarm Optimization.

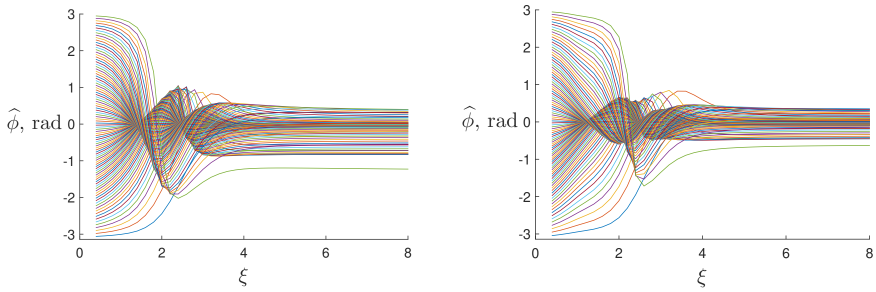

Fig. 2 Phase deviation of particles from synchronous one

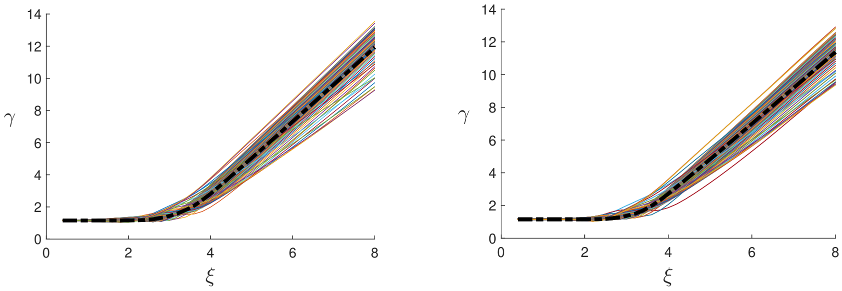

Fig. 3 Reduced energy of particles

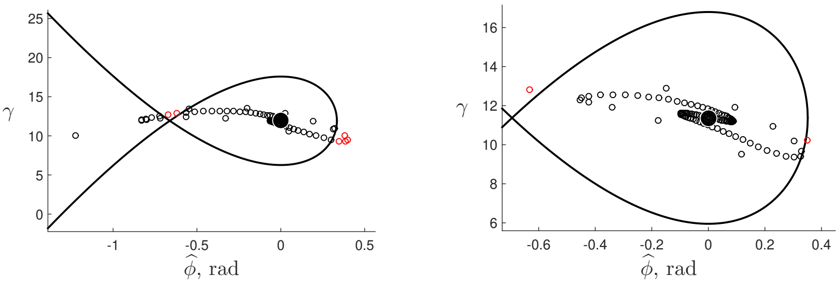

Fig. 4 Separatrix and phase-energy distribution of particles at accelerator exit

4 Conclusion

The paper presents an approbation of a genetic stochastic algorithm with adaptation of a covariance matrix on a multi-extreme large-dimensional problem, namely, the problem of beam dynamics optimization in linear accelerator. The practice of numerical experiments and the results obtained indicate the simplicity, convenience and effectiveness of this method. The results of successive optimization show beam quality improvement.

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