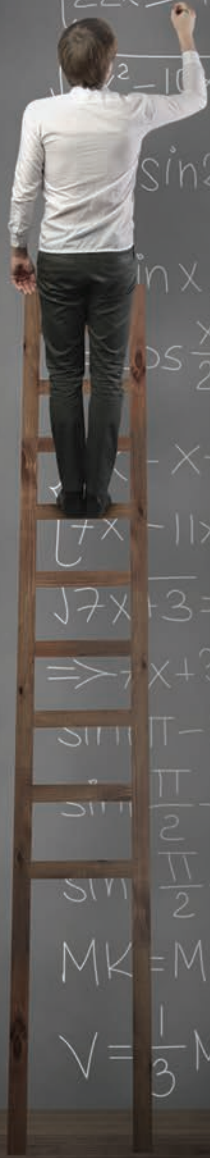
4th International Conference on Matrix Methods in Mathematics and Applications (MMMA-2015)

August 24-28, 2015

matrix.inm.ras.ru/mmma-2015/



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MMMA-2015

WE ARE DELIGHTED TO ANNOUNCE THE

4th International Conference on Matrix Methods in Mathematics and Applications, MMMA-2015, which will be held from 24th to 28th August, 2015 at Skolkovo Institute of Science and Technology, located near Moscow in Russia. Sunday, August 23 and Saturday, August 30. During this conference we will have a special event in honor of 60-th anniversary of Prof. Eugene Tyrtyshnikov.

INVITED SPEAKERS INCLUDE:

Chistoph Schwab (ETH Zurich)

Andrzej Cichocki (Brain Science Institute, RIKEN Lab)

Gunnar Martinsson (UC Boulder)

Paul Van Dooren (UCLouivan)

Jacob White (MIT)

Denis Zorin (NYU Courant)

André Uschmajew (Uni. Bonn)

Sergey Kabanikhin (ICM MG SO RAS)

Valery Ilin (ICM MG SO RAS)

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The conference is sponsored by Huawei company

4th International Conference on Matrix Methods in Mathematics and Applications (MMMA-2015)

August 24-28, 2015

Program

August 24, Monday

9.00 - 9.30	Registration and Greetings	
9.30 - 10.10	N.Krivulin	
10.10 - 10.50	P.G.Martinsson	
10.50 - 11.30	A.Cichocki	
11.30 - 12.00	Coffee break	
12.00 - 12.30	M.J. de la Puente	A.Novikov
12.30 - 13.00	S.Sergeev	S.Suetin
13.00 - 13.30	O.Markova	M.Botchev
13.30 - 14.00	Mikhail Khrystik	V.Kazeev
14.00 - 15.00	Lunch	
15.00 - 15.40	J.K.White	
15.40 - 16.20	Ch.Schwab	
16.20 - 17.00	V.Protasov	
17.00 - 17.30	Coffee break	
17.30 - 18.00	N.Costa	S.Dolgov
18.00 - 18.30	A.Voynov	M.Litsarev
18.30	Transfer	

August 25, Tuesday

9.00 - 9.30	Morning coffee		
9.30 - 10.10	M.Thess		
10.10 - 10.50	K.Vorontsov		
10.50 - 11.30	V.Lyashev		
11.30 - 12.00	Coffee break		
12.00 - 12.30	B.Bakhadly	H.Walach	A.Fonarev
12.30 - 13.00	N.Ostroukhova	S.Goreinov	E.Frolov
13.00 - 13.30	S.Furtado	D.Zheltkov	L.Muravleva
13.30 - 14.30	Lunch		
14.30 - 16.30	Special event for E.E. Tyrtyshnikov		
16.30 - 19.30	Banquet		
20.00	Transfer		

August 26, Wednesday

9.00 - 9.30	Morning coffee		
9.30 - 10.10	V.Ilin		
10.10 - 10.50	I.Sofronov		
10.50 - 11.30	P.Vabishchevich		
11.30 - 12.00	Coffee break		
12.00 - 12.30	V.Chugunov	A.Tsupak	
12.30 - 13.00	A.Abdikalykov	O.Ergül	
13.00 - 13.30	C. Di Fiore	A.Shapeev	
13.30 - 14.00	T.Saluev	M.Rakhuba	
14.00 - 15.00	Lunch		
15.00 - 15.40	D. Zorin		
15.40 - 16.20	P. Van Dooren		
16.20 - 17.00	Y. Shitov		
17.00 - 17.30	Coffee break		
17.30 - 18.00	I. Kaporin	S. Stavtsev	A.Mennouni
18.00 - 18.30	A.Maksaev	A.Mikhalev	A.Aoufi
18.30	Transfer		

August 27, Thursday

9.00 - 9.30	Morning coffee
9.30 - 10.10	D.O.Logofet
10.10 - 10.50	N.Mastronardi
10.50 - 11.30	A.Uschmajew
11.30 - 12.10	V.Sergeichuk
12.10 - 13.00	Lunch
13.00	Free Time

August 28, Friday

9.00 - 9.30	Morning coffee		
9.30 - 10.10	S.Kabanikhin		
10.10 - 10.50	Y.Eidelman		
10.50 - 11.30	L.Knizhnerman		
11.30 - 12.00	Coffee break		
12.00 - 12.30	M.Efimov	S.Wolf	D.Sushnikova
12.30 - 13.00	A.Guterman	S.Cipolla	E.Muravleva
13.00 - 13.30	M.Budrevich	A.Chertkov	D.Bianchi
13.30 - 14.00	V.Kushel	-	A.Rodomanov
14.00 - 15.00	Lunch		
15.00 - 15.40	M.Akian		
15.40 - 16.20	A.Yagola		
16.20 - 17.00	A.Aptekarev		
17.00 - 17.30	Coffee break		
17.30 - 18.00	M.Tyaglov	S.Matveev	V.Lebedev
18.00 - 18.30	K.Taranin	D.Kolesnikov	I.Ostanin
18.30	Transfer		

Talks:
- Plenary talks
- Section talks
- Shared section

Meals:
- Coffee break
- Lunch
- Banquet

Activity:
- Special event
- Free Time
- Transfer

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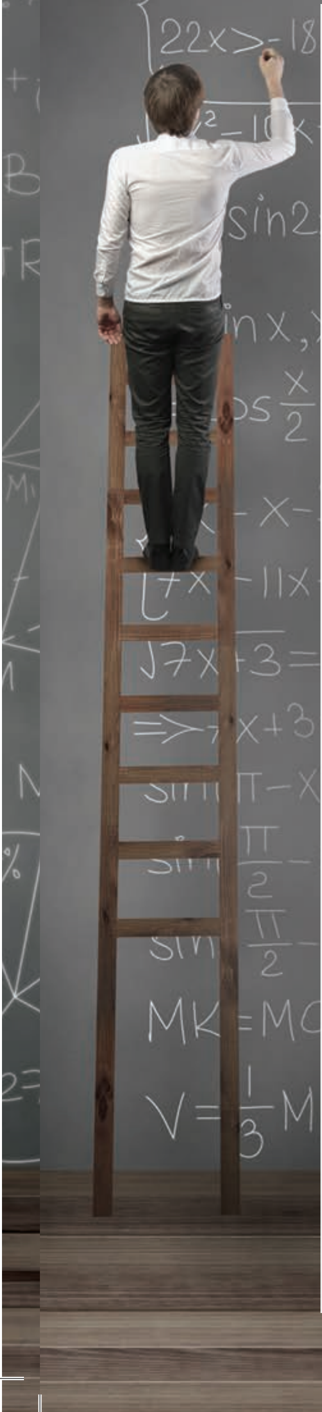


August 24, Monday

9.00 - 9.20	Registration
9.20 - 9.30	Greetings
9.30 - 10.10	**Nikolai Krivulin** *Solution of a Tropical Optimization Problem Using Matrix Sparsification*
10.10 - 10.50	**Per-Gunnar Martinsson** *Randomized methods for accelerating matrix factorizations and structured matrix computations*
10.50 - 11.30	**Andrzej Cichocki** *Tensor/Matrix Decompositions and Tensor Networks for Linked Multiway Component Analysis and Big Data Processing*
11.30 - 12.00	Coffee break
12.00 - 12.30	**María Jesús de la Puente** *Matrices commuting with a given normal tropical matrix*
12.00 - 12.30	**Alexander Novikov, Dmitry Podoprikhin, Anton Osokin, Dmitry Vetrov** *Tensorizing Neural Networks*
12.30 - 13.00	**Sergey Sergeev, Ján Plavka** *Characterizing matrices with X-simple image eigenspace in tropical semirings*
12.30 - 13.00	**Sergey Suetin** *Limit zero distribution of Hermite–Padé polynomials and efficient analytic continuation of power series*
13.00 - 13.30	**Olga Markova, Alexander Guterman, Volker Mehrmann** *Lengths of quasi-commutative pairs of matrices*
13.00 - 13.30	**Mikhail Botchev** *A nested Schur approach for mesh-independent preconditioning in time-domain photonics modeling*
13.30 - 14.00	**Vladimir Kazeev, Christoph Schwab** *Quantized tensor-structured finite elements for secondorder elliptic PDEs in two dimensions*
14.00 - 15.00	Lunch
15.00 - 15.40	**Jacob K. White, Jorge Villena, Athanasios Polymeridis, Luca Daniel, Richard Zhang** *Voxelized Geometries, Volume Integral Equations, and the Reemergence of FFT-based Sparsification*
15.40 - 16.20	**Christoph Schwab** *Covariance regularity and H-matrix approximation for rough random fields*
16.20 - 17.00	**Vladimir Protasov** *Spectral simplex method*
17.00 - 17.30	Coffee break
17.30 - 18.00	**Natalia Costa** *Non-Hermitian Hamiltonians in Quantum Mechanics*
17.30 - 18.00	**Sergey Dolgov** *On parallelization of alternating linear schemes for lowrank high-dimensional optimization*
18.00 - 18.30	**Andrey Voynov** *Invariant polyhedra of linear operators and the Černy*
18.00 - 18.30	**Mikhail Litsarev, Ivan Oseledets** *"Inflated domain" method for the Schrödinger equation in free space*
18.30	Transfer

4th International Conference on Matrix Methods in Mathematics and Applications (MMMA-2015)

August 24-28, 2015

August 25, Tuesday

9.00 - 9.30	Morning coffee
9.30 - 10.10	**Michael Thess** *Approximation Framework for Combined Recommendations and Price Optimization*
10.10 - 10.50	**Konstantin Vorontsov** *Regularized Matrix Factorization for Topic Modeling of Text Collections*
10.50 - 11.30	**Vladimir Lyashev** *Mathematical Problems in Wireless Communications*
11.30 - 12.00	Coffee break
12.00 - 12.30	**Bakhad Bakhadly** *Orthogonality graphs of subsets of matrices with special properties*
12.00 - 12.30	**Hanna Walach, Emil Kieri, Christian Lubich** *Discretized dynamical low-rank approximation in thepresence of small singular values*
12.00 - 12.30	**Alexander Fonarev** *Rectangular Maximal-Volume Concept for Cold Start Problem in Collaborative Filtering*
12.30 - 13.00	**Natasha Ostroukhova** *Hamiltonian sets of polygonal paths in simple assembly graphs*
12.30 - 13.00	**Sergei Goreinov** *Kolmogorov's criterion and best C-norm multivariate approximation*
12.30 - 13.00	**Evgeny Frolov, Ivan Oseledets** *Tensor decompositions in recommender systems - how and why it works*
13.00 - 13.30	**Susana Furtado, Maria Isabel Bueno, Froilan Dopico** *Linearizations of Hermitian matrix polynomials preserving the sign characteristic*
13.00 - 13.30	**Dmitry Zheltkov, Eugene Tyrtyshnikov** *Global optimization based on tensor train representation*
13.00 - 13.30	**Larisa Muravleva** *A bi-projection method for numerical modeling of viscoplastic Bingham medium flows*
14.00 - 14.30	Lunch
14.30 - 16.30	Special event for E.E. Tyrtyshnikov
16.30 -19.30	Banquet
20.00	Transfer

August 26, Wednesday

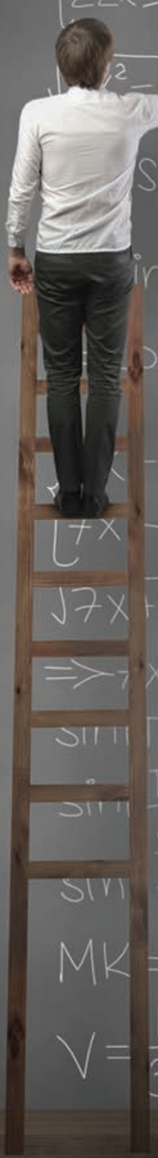
9.00 - 9.30	Morning coffee
9.30 - 10.10	**Valery Ilin** *On the Multi-Preconditioned Methods in Krylov Subspaces*
10.10 - 10.50	**Ivan Sofronov, Nikolai Zaitsev** *Transparent boundary conditions for anisotropic transversally heterogeneous media*
10.50 - 11.30	**Petr Vabishchevich** *Numerical solution of problems for a space-fractional diffusion equation*
11.30 - 12.00	Coffee break
12.00 - 12.30	**Vadim Chugunov, Khakim Ikramov** *A complete solution of the permutability problem for Toeplitz and Hankel matrices*
12.00 - 12.30	**Aleksei Tsupak, Yuri Smirnov, Mikhail Medvedik** *1D+2D+3D Diffraction Problems: Integral Equations, Matrices, and Applications*
12.30 - 13.00	**Abdikozha Abdikalykov, Khakim Ikramov** *Unitary automorphisms of the spaces of Toeplitz, Hankel, and (T+H)-matrices*
12.30 - 13.00	**Özgür Ergül, Barışcan Karaosmanoğlu, Can Önol** *Electromagnetic Optimizations Using Heuristic Algorithms Combined With the Ordinary and Approximate Forms of the Multilevel Fast Multipole Algorithm*
13.00 - 13.30	**Carmine Di Fiore, Francesco Tudisco, Paolo Zellini** *Lower triangular Toeplitz systems and the sequence of Bernoulli numbers*
13.00 - 13.30	**Alexander Shapeev** *Perspectives of low rank approximation in molecular modelling*
13.30 - 14.00	**Tigran Saluev** *Structure of operator norms*
13.30 - 14.00	**Maxim Rakhuba, Ivan Oseledets** *Grid-based electronic structure calculations: the tensor decomposition approach*
14.00 - 14.30	Lunch
15.30 - 15.40	**Denis Zorin**
15.40 – 16.20	**Paul Van Dooren** *Generalized Fiedler pencils and their numerical stability*
16.20 – 17.00	**Yaroslav Shitov** *Nonnegative rank depends on the field*
17.30 – 18.00	**Igor Kaporin** *Deterministic bounds on the restricted isometry in compressed sensing matrices*
17.30 – 18.00	**Stanislav Stavtsev, Vadim Chugunov** *Block LU preconditioner for some integral equation*
17.30 – 18.00	**Abdelaziz Mennouni** *On numerical range and numerical radius for some integral operators*
18.00 – 18.30	**Artem Maksaev** *The characterization of operators preserving scrambling index for matrices over semirings*
18.00 – 18.30	**Alexander Mikhalev** *Rectangular submatrices of maximum volume and its applications*
18.00 – 18.30	**Asdin Aoufi, Gilles Damamme** *Parallel explicit/implicit computing of multidimensional TiC combustion synthesis using openmp*
18.30	Transfer

August 27, Thursday

9.00 - 9.30	Morning coffee
9.30 - 10.10	**Dmitrii O. Logofet** *Current Problems in Matrix Population Models: Expanding Classics and New Discoveries*
10.10 - 10.50	**Nicola Mastronardi, Paul Van Dooren** *QR algorithm, blurred shifts and the computation of the Jordan structure of an eigenvalue*
10.50 - 11.30	**Andre Uschmajew** *On descent algorithms on low-rank matrix varieties*
11.30 - 12.10	**Vladimir V. Sergeichuk** *Classification problems for systems of forms and linear mappings*
12.10 - 13.00	Lunch
13.00	Time free

August 28, Friday

9.00 - 9.30	Morning coffee
9.30 - 10.10	**Sergey Kabanikhin, Nikita Novikov, Ivan Oseledets, Maxim Shishlenin** *Toeplitz matrices in multidimensional inverse problems*
10.10 - 10.50	**Yuli Eidelman, Iulian Haimovici** *Eigenvalue algorithms for Hermitian quasiseparable of any order matrices*
10.50 - 11.30	**Leonid Knizhnerman, Vladimir Druskin, Stefan Güttel** *Subdiagonal rational approximation of the function $z^{-1/2}$ on the union of a positive and a negative real line segment*
11.30 - 12.00	Coffee break
12.00 - 12.30	**Mikhail Efimov** *Monotone maps on matrix and operator algebras with respect to sharp partial order*
12.00 - 12.30	**Daria Sushnikova** *Fast block low-rank direct solver for sparse matrices*
12.00 - 12.30	**Sebastian Wolf, Benjamin Huber, Reinhold Schneider** *A Low-Rank Optimization Algorithm based on the Projector Splitting Integrator*
12.30 - 13.00	**Alexander Guterman** *Extremal matrix centralizers and their applications*
12.30 - 13.00	**Stefano Cipolla, Carmine Di Fiore** *Adaptive matrix algebras in unconstrained optimization*
12.30 - 13.00	**Ekaterina Muravleva, Ivan Oseledets** *Fast low-rank solution of the Poisson equation based on cross approximation*
13.00 - 13.30	**Mikhail Budrevich, Gregor Dolinar, Bojan Kuzma, Alexander Guterman** *Fully indecomposable non-convertible (0,1)-matrices*
13.00 - 13.30	**Mikhail Budrevich1, Gregor Dolinar, Bojan Kuzma, Alexander Guterman** *Fully indecomposable non-convertible (0,1)-matrices*
13.00 - 13.30	**Davide Bianchi, Marco Donatelli, Yuantao Cai, Ting-Zhu Huang** *Regularization preconditioners for frame-based image deblurring with reduced boundary artifacts*
13.30 - 14.00	**Volha Kushel** *On rank-one perturbations of stable matrices*
13.30 - 14.00	**Anton Rodomanov, Alexander Novikov, Anton Osokin, Dmitry Vetrov** *Probabilistic graphical models: a tensorial perspective*
13.30 - 14.00	**Andrei Chertkov, Ivan Oseledets** *Tensor train approximation in active subspace variables with application to parametric partial differential equations.*
14.00 - 14.30	Lunch
15.30 - 15.40	**Marianne Akian** *Majorization inequalities for valuations of eigenvalues using tropical algebra*
15.40 - 16.20	**Anatoly Yagola** *Error Estimation for Ill-Posed Problems*



August 28, Friday

16.20 - 17.00	**Alexander Aptekarev** *Spectra of Random Matrices and Riemann -Hilbert Problems for Matrix Valued Analytic Functions*
17.30 - 18.00	**Sergey Matveev** *Parallel implementation of the fast method solving the kinetic equations of aggregation and fragmentation*
17.30 - 18.00	**Mikhail Tyaglov, Olga Kushel, Olga Holtz, Sergei Khrushchev** *Criterion of total positivity of generalized Hurwitz matrices*
18.00 - 18.30	**Vadim Lebedev** *Speeding-up Convolutional Neural Networks Using Finetuned CP-Decomposition*
18.00 - 18.30	**Konstantin Taranin** *On attainability of values for permanents of (0,1) matrices*
18.00 - 18.30	**Denis Kolesnikov** *From low-rank approximation to an efficient rational Krylov subspace method for the Lyapunov equation*
18.00 - 18.30	**Igor Ostanin** *Towards fast topological shape optimization with boundary elements*
18.30	Transfer

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LOCATION: Skolkovo Innovation Centre, Nobel Street, building 3, Moscow

CONFERENCE VENUE: 3rd floor, Multifunction Area; 1st floor Auditorium #148

TRANSFER:

1. Morning transfer from Marriott Grand Hotel Moscow (26/1 Tverskaya Street) to Technopark Skolkovo leaves at 8:15 a.m. stopping at the Moscow School of Management Skolkovo at 8:45 a.m.

   Evening transfer to Marriott Grand leaves at 6:30 p.m. stopping at the Moscow School of Management Skolkovo at 6:40 p.m.
2. Morning transfer from Park Pobedy subway station (Kutuzovsky prospekt, 38, see the map below) leaves at 8:30 a.m.

   Evening transfer to Park Pobedy subway station leaves at 6:30 p.m.

IMPORTANT:

On Thursday, August 27, return transfers leaves at 1:15 p.m.

IF YOU PLAN TO ARRIVE BY CAR:

1. Skolkovskoye shosse to «P3» Parking. 7-8 minutes from Moscow Ring Road 730 guest parking places
2. Minskoye shosse to «Polyot». 30-40 minutes from Moscow Ring Road 50 guest parking places, on written request
3. From parking places Skolkovo minibuses will take you to the offices.

Please send the car registration number to events@skoltech.ru before August, 21 to obtain a pass.

Please take notice that the Parking lot #3 costs 50 rubles per hour.

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Wi-fi access:
Skoltech_GUEST, no password required

Lunch:
1st floor, Nobel Street, building 5, Cafeteria 'Daily Food'

Reception:
August, 25, 4th floor, Cohort Space, 4:30 p.m.

Monday, 09:30 – 10:10

Solution of a Tropical Optimization Problem Using Matrix Sparsification

Nikolai Krivulin
Saint Petersburg State University

A multidimensional optimization problem, which arises in various applications in the form of minimization of span seminorm is considered in the framework of tropical (idempotent) mathematics. The problem is formulated to minimize a nonlinear function, which is defined on vectors over an idempotent semifield, given by a matrix, and calculated using multiplicative conjugate transposition. To solve the problem, we apply and develop methods of tropical optimization proposed and investigated in [1-3]. First, we find the minimum value of the objective function, and give a partial solution as a subset of vectors represented in an explicit form. We characterize all solutions to the problem by a system of simultaneous vector equation and inequality, and use this characterization to investigate properties of the solution set, which, in particular, turns out to be closed under vector addition and scalar multiplication. Furthermore, a matrix sparsification technique is developed to drop, without affecting the solution of the problem, those entries in the matrix which are below prescribed threshold values. By combining this technique with the above characterization, the previous partial solution is extended to a wider solution subset, and then to a complete solution described as a family of subsets. Finally, we offer a backtracking procedure that generates all members of the family, and derive an explicit representation for the complete solution in a compact vector form.

References

[1] Krivulin, N. A constrained tropical optimization problem: Complete solution and application example. In G. L. Litvinov and S. N. Sergeev (eds.) Tropical and Idempotent Mathematics and Applications, Contemp. Math., vol. 616, Providence, RI: AMS, 2014, pp. 163–177.
[2] Krivulin, N. A multidimensional tropical optimization problem with nonlinear objective function and linear constraints. Optimization, vol. 64, no. 5, pp. 1107–1129, 2015.
[3] Krivulin, N. Extremal properties of tropical eigenvalues and solutions to tropical optimization problems. Linear Algebra Appl., vol. 468, pp. 211–232, 2015.

Monday, 10:10 – 10:50

Randomized methods for accelerating matrix factorizations and structured matrix computations

Per-Gunnar Martinsson
University of Colorado

Methods based on randomized projections have over the last several years proven to be powerful tools for computing low-rank approximations to matrices whose singular values exhibit appropriate decay. In this talk, we describe randomized algorithms for several different environments, including approximation of so called ”rank-structured” matrices, for which only certain off-diagonal blocks (not the matrix itself) admit accurate low-rank approximations. Matrices of this type often arise in the construction of O(N) direct solvers for elliptic PDEs.

Monday, 10:50 – 11:30

Tensor/Matrix Decompositions and Tensor Networks for Linked Multiway Component Analysis and Big Data Processing

Andrzej Cichocki
RIKEN Brain Science Institute

In this talk we present several multi-block matrix and tensor decomposition models in emerging applications in computational neuroscience. Fundamental tensor and matrix decomposition models are discussed that allow to extract physically meaningful common and statistically independent components from massive large-scale data sets. Basic Tensor Train (TT) network models for very large scale problems related to computation of PCA/SVD, generalized eigenvalue decompositions (GEVD) and sparse canonical correlation analysis will be also briefly presented.

Monday, 12:00 – 12:30

Matrices commuting with a given normal tropical matrix

María Jesús de la Puente, Jorge Linde
Universidad Complutense

Consider the space M_n^{nor} of square normal matrices $X = (x_{ij})$ over $\mathbb{R} \cup \{-\infty\}$, i.e., $-\infty \leq x_{ij} \leq 0$ and $x_{ii} = 0$. Endow M_n^{nor} with the tropical sum $\oplus$ and multiplication $\odot$. Fix a real matrix $A \in M_n^{nor}$ and consider the set $\Omega(A)$ of matrices in M_n^{nor} which commute with A. We prove that $\Omega(A)$ is a finite union of alcoved polytopes; in particular, $\Omega(A)$ is a finite union of convex sets. The set $\Omega^A(A)$ of matrices X such that $A \odot X = X \odot A = A$ is also a finite union of alcoved polytopes. The same is true for the set $\Omega'(A)$ of matrices X such that $A \odot X = X \odot A = X$.

A topology is given to M_n^{nor}. Then, the set $\Omega^A(A)$ is a neighborhood of the identity matrix I. If A is strictly normal (i.e., $-\infty \leq x_{ij} < 0$ if $i \neq j$), then $\Omega'(A)$ is a neighborhood of the zero matrix. In some cases, $\Omega(A)$ is a neighborhood of A.

We explore the relationship between the polyhedral complexes $spanA$, $spanX$ and $span(AX)$, when A and X commute.

We produce examples of matrices which commute, in any dimension.

This is joint work with J. Linde.

References

[1] J. Linde and M.J. de la Puente; Matrices commuting with a given normal tropical matrix; to appear in Linear Algebra and Its Applications DOI: 10.1016/j.laa.2015.04.032

Monday, 12:00 – 12:30

Tensorizing Neural Networks

Alexander Novikov[1], Dmitry Podoprikhin[2], Anton Osokin[3], Dmitry Vetrov[4]
[1] *Skolkovo Institute of Science and Technology*
[2] *Lomonosov Moscow State University*
[3] *SIERRA, INRIA*
[4] *National Research University Higher School of Economics and Skolkovo Institute of Science and Technology*

Deep neural networks currently demonstrate state-of-the-art performance in several domains. At the same time, models of this class are very demanding in terms of computational resources. In particular, a large amount of memory is required by fully-connected layers which are used in a lot of different architectures. This makes it hard to use these models on low-end devices and stops the further increase of the model size. We propose a compact parametrization for fully-connected layers. Our approach consists in representing the dense weight matrix of the layer in the Tensor Train format [1]. This format allows to reduce the number of parameters by a huge factor while keeping the expressive power of the layer. In particular, for the Very Deep VGG networks [2] we report the compression of the dense weight matrix of a fully-connected layer up to 200 000 times leading to the compression of the whole network up to 7 times.

References

[1] I. V. Oseledets, "Tensor-Train decomposition," *SIAM J. Scientific Computing*, vol. 33, no. 5, pp. 2295–2317, 2011

[2] K. Simonyan and A. Zisserman, "Very deep convolutional networks for large-scale image recognition," *International Conference on Learning Representations (ICLR)*, 2015

Monday, 12:30 – 13:00

Characterizing matrices with *X*-simple image eigenspace in tropical semirings

Sergey Sergeev[1], Ján Plavka[2]

[1] *University of Birmingham*

[2] *The Technical University of Košice, Department of Mathematics and Theoretical Informatics*

A matrix A is said to have X-simple image eigenspace if any eigenvector x belonging to the interval $X = \{x\colon \underline{x} \leq x \leq \overline{x}\}$ is the unique solution of the system $A \otimes y = x$ in X. Our purpose is to characterize such matrices in the linear algebra over max-min and max-plus semirings or suggest a method that verifies whether A has X-simple image eigenspace.

The characterized property is related to and motivated by the notions of simple image set and weak robustness (or weak stability) that have been studied in tropical algebras, as well as by a desire to develop a tropical counterpart of the interval analysis.

Monday, 12:30 – 13:00

Limit zero distribution of Hermite–Padé polynomials and efficient analytic continuation of power series

Sergey Suetin

Steklov Mathematical Institute of the Russian Academy of Sciences

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The main purpose of our talk is to explain the use of Hermite–Padé polynomials in constructive approximation.

Let f be a multivalued analytic function given by the representation

$$f(z) = \prod_{j=1}^{p}(z-a_j)^{\alpha_j}, \quad \text{where} \quad \alpha_j \in \mathbb{C}\setminus\mathbb{Z}, \quad \sum_{j=1}^{p}\alpha_j = 0, \tag{1}$$

and all the points $a_j \in \mathbb{C}$ supposed to be distinct from each other, i.e., $a_j \neq a_k$ when $j \neq k$. Thus the function f is a multivalued analytic function in the domain $\mathbb{C}\setminus\Sigma$, where $\Sigma = \{a_1,\dots,a_p\}$ is a finite set of branch points of f. Let fix the germ of f by the condition $f(\infty) = 1$.

For a fixed $n \in \mathbb{N}$ let $P_{n,0}, P_{n,1} \in \mathbb{C}_n[z]\setminus\{0\}$ be the Padé polynomials for the fixed germ of f, i.e.,

$$(P_{n,0}\cdot 1 + P_{n,1}f)(z) = O\Big(\frac{1}{z^{n+1}}\Big), \quad z \to \infty. \tag{2}$$

Then by the seminal Stahl's Theorem (1985–1986) we obtain that

$$-\frac{P_{n,0}}{P_{n,1}}(z) \overset{\text{cap}}{\longrightarrow} f(z), \quad n \to \infty, \quad z \in D, \tag{3}$$

where D is the so-called Stahl's 'maximal' domain of convergence. The boundary $S = \partial D = \overline{\mathbb{C}}\setminus D$ consists of finite number of analytic arcs and possesses some property of 'symmetry'.

Let suppose now that in (1) we have $2\alpha_j \in \mathbb{C}\setminus\mathbb{Z}$. Then it follows that the functions $1, f, f^2$ are rationally independent. Let define type I Hermite–Padé polynomials $Q_{n,0}, Q_{n,1}, Q_{n,2} \in \mathbb{C}_n[z]\setminus\{0\}$ by the relation (cf. (2))

$$(Q_{n,0}\cdot 1 + Q_{n,1}f + Q_{n,2}f^2)(z) = O\Big(\frac{1}{z^{2n+2}}\Big), \quad z \to \infty. \tag{4}$$

A very natural question arises: what can we say about the ratio $-Q_{n,1}/Q_{n,2}$ (cf. (3)) ? Does it converge to an analytic function which is connected with the given f or not? If so, does the sequence $H_n(z) := -Q_{n,1}/Q_{n,2}$ might give us some more information about the analytic properties of the function f than the sequence of Padé approximants $[n/n]_f = -P_{n,0}/P_{n,1}$ usually does? In general the answer is unknown but in some special cases the answer is positive and very interesting (see [1], [2], [3], [4]). It turns out that the rational functions H_n possesses the properties which make them very close to the best Chebyshev approximants. Namely all n zeros and n poles of H_n are not prescribed but free and also it has near $2n$ nodes of free interpolation of the given analytic function f. Zeros and poles of H_n and the nodes as well have some limit distributions as $n \to \infty$, which solve theoretic potential extremal problem. As well as in Stahl's Theorem (see (3)), the rational functions H_n itself converge in capacity to another branch of the given function f. It is proved that in some partial cases the functions H_n possesses 'almost Chebyshev' alternation property.

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Monday, 13:00 – 13:30

Lengths of quasi-commutative pairs of matrices

Olga Markova[1], Alexander Guterman[1], Volker Mehrmann[2]

[1] *Lomonosov Moscow State University*

[2] *Technische Universität Berlin*

Let $\mathbb{F}$ be a field and let $\mathcal{A}$ be a finite-dimensional associative $\mathbb{F}$-algebra. Following [1, 2], we define *the length of a finite system of generators* $\mathcal{S}$ of a given algebra $\mathcal{A}$ as the smallest number k such that words in $\mathcal{S}$ of length not greater than k generate $\mathcal{A}$ as a vector space, and *the length of the algebra* is the maximum of the lengths of its systems of generators. The length function has applications in computing methods of the mechanics of isotropic continua [3].

For the full matrix algebra $M_n(\mathbb{F})$ the problem of computing the length as a function of the matrix size n is studied since 1984 and is still open. However, there exist some good bounds for the lengths of matrix sets satisfying some additional conditions. In this talk we discuss the length evaluation problem for quasi-commuting pairs of matrices (we say that A, B in $M_n(\mathbb{F})$ *quasi-commute* if AB and BA are linearly dependent). The quasi-commutativity is an important relation in quantum physics [4, 5].

Among all quasi-commuting pairs of matrices there are two special classes that can be singled out and where the analysis is easier than for general pairs. These are (I) commuting pairs and (II) quasi-commutative, non-commuting pairs with a nilpotent product.

We show that in each of these cases $l(\mathcal{S}) \leq n-1$ and, moreover, for any $l = 1, \dots, n-1$, each these two classes contains a pair of matrices with length l.

If a quasi-commuting pair $\mathcal{S} = \{A, B\} \subset M_n(\mathbb{F})$ does not belong to the class (I)∪(II), then $AB = \varepsilon BA$ where the commutativity factor ε is a primitive k-th root of unity for some $k \leq n$. We will show that in this case the situation is very different from the commutative and nilpotent case. We obtain sharp bounds

$$2k - 2 \leq l(\mathcal{S}) \leq 2n - 2.$$

Moreover, we show that the length $2n-2$ is attainable if and only if the quasi-commutativity factor is a primitive n-th root of unity. For roots of unity with other degrees we provide a sharp upper bound

$$l(\{A, B\}) \leq \max\{2(n-k) - r; n + k\} - 2,$$

where r denotes the algebraic multiplicity of 0 as an eigenvalue of AB.

For the length realizability problem of quasi-commuting matrix pairs not belonging to the class (I)∪(II) we have the following partial results:
1) All even numbers between 2 and $2n-2$ are realizable;
2) The numbers 1 and $2n-3$ are not realizable;
3) The number $2n-5$ is realizable for $n = 4, 6$ and is not realizable for $n = 5$ or $n > 6$;
4) For all natural numbers k, m, n, $k \geq 2$, $n \geq (m+1)k$ there exist matrices $A_n, B_n \in M_n(\mathbb{F})$ such that A_n and B_n quasi-commute with the factor being a primitive k-th root of unity and $l(\{A_n, B_n\}) = (m+1)k - 1$.

The talk is based on our paper [6].

The work was partially supported by grants MD-962.2014.1, RFBR 13-01-00234-a and 15-31-20329.

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Monday, 13:00 – 13:30

A nested Schur approach for mesh-independent preconditioning in time-domain photonics modeling

Mikhail Botchev
University of Twente

A standard tool for time-domain photonic crystal modeling is the so-called finite-difference time-domain (FDTD) method. It is usually based on a staggered central difference approximation of the time dependent Maxwell equations in space and time. A significant reduction in CPU times typically required by the FDTD method can be achieved by replacing the time integration in FDTD by exponential rational Krylov subspace schemes (see, e.g., [2]). This means that the spatial discretization of the Maxwell equations is carried out first and leads to a initial-value problem (IVP) for a system of ordinary differential equations

$$y'(t) = -\mathcal{A}y(t) + g(t), \qquad y(0) = v,$$

where $\mathcal{A}$ is the spatially discretized Maxwell operator. In our setting, the Maxwell equations are solved with special non-reflecting boundary conditions, namely, the so-called PML (perfectly matched layer) conditions. Therefore, the vector function $y(t)$ contains the mesh values of the electric and magnetic fields and the auxiliary PML variables. Due to the PML boundary conditions, the matrix $\mathcal{A}$ is not (similar to) a skew-symmetric matrix. The IVP system is then solved by a rational Krylov subspace method, such as one described in [1].

The rational Krylov subspace methods require solution of large sparse linear systems with the matrices $I + \gamma\mathcal{A}$, with $\gamma > 0$ given. We discuss a Schur complement approach to construct robust preconditioners for solving these linear systems iteratively. We show that the approach leads to a preconditioner for which the iteration number does not grow as the spatial mesh is refined. The proposed mesh-independent preconditioners are shown to outperform more standard preconditioners such as based on the field splitting or on the alternative direction implicit (ADI) splitting.

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Monday, 13:30 – 14:00

Quantized tensor-structured finite elements for second-order elliptic PDEs in two dimensions

Vladimir Kazeev, Christoph Schwab
Seminar for Applied Mathematics, ETH Zurich

We analyze a first-order, h-version finite-element (FE) approximation of a second order elliptic model problem in a curvilinear polygon whose solutions have point singularities but belong to a countably normed space of analytic functions. The FE solution is well known to converge with algebraic rate at most 1/2 in terms of the number of degrees of freedom, and even slower due to the singularities. This, however, turns out to be irrelevant when the FE coefficient vector is represented in the so-called *quantized tensor train* format. We prove that such FE approximations converge exponentially in terms of the effective number N of degrees of freedom involved in the representation: $N = \mathcal{O}(\log^5 \varepsilon^{-1})$, where ε is the accuracy measured in the energy norm.

Numerically we show for solutions from the same class that the entire process of solving the tensor-structured Galerkin first-order FE discretization can achieve accuracy ε in the energy norm with $N = \mathcal{O}(\log^\kappa \varepsilon^{-1})$ parameters, where $\kappa < 3$.

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Monday, 15:00 – 15:40

Voxelized Geometries, Volume Integral Equations, and the Reemergence of FFT-based Sparsification

Jacob K. White, Jorge Villena, Athanasios Polymeridis, Luca Daniel, Richard Zhang
Massachusetts Institute of Technology

In disciplines as diverse as microfluidics, nanotechnology, medical imaging, and 3-D printing, model generation must be completely automatic, yet extremely reliable. This combination has favored model generation based on the most-easily-manipulated geometric element, the cube. When volume integral equations are used to analyze cube-based models (referred to as "pixelized" in 2-D and "voxelized" in 3-D), the result is often a multidimensional Toeplitz-plus-Hankel (TPH) matrix, well-known to be easily sparsified with the FFT. In this talk we will describe several recent applications of volume integral equations applied to voxelized models, and describe some of the newer challenges and insights, such as using numerically-computed TPH approximations to non-TPH problems. We will also note a reemerging difficulty, preconditioning multidimensional TPH matrices, and make note of the seminal insights of Professsor Eugene Tyrtyshnikov, on his 60th anniversary.

Monday, 15:40 – 16:20

Covariance regularity and H-matrix approximation for rough random fields

Christoph Schwab
ETH Zurich

In an open, bounded domain $\mathrm{D} \subset \mathbb{R}^n$ with smooth boundary $\partial\mathrm{D}$ or on a smooth, closed and compact, Riemannian n-manifold $\mathcal{M} \subset \mathbb{R}^{n+1}$, we consider the linear operator equation $Au = f$ where A is a boundedly invertible, strongly elliptic pseudodifferential operator of order $r \in \mathbb{R}$ with analytic coefficients, covering all linear, second order elliptic PDEs as well as their boundary reductions. Here, $f \in L^2(\Omega; H^t)$ is an H^t-valued random field with finite second moments, with H^t denoting the (isotropic) Sobolev space of (not necessarily integer) order t modelled on the domain D or manifold $\mathcal{M}$, respectively. We prove that the random solution's covariance kernel $K_u = (A^{-1} \otimes A^{-1})K_f$ on $\mathrm{D}\times\mathrm{D}$ (resp. $\mathcal{M}\times\mathcal{M}$) is an asymptotically smooth function provided that the covariance function K_f of the random data is a Schwartz distributional kernel of an elliptic pseudodifferential operator. As a consequence, numerical $\mathcal{H}$-matrix calculus allows deterministic approximation of singular covariances K_u of the random solution $u = A^{-1} f \in L^2(\Omega; H^{t-r})$ in $\mathrm{D} \times \mathrm{D}$ with work versus accuracy essentially equal to that for the mean field approximation in D, overcoming the curse of dimensionality in this case.

Joint work with J. Dölz and H. Harbrecht (University of Basle, Switzerland).

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Monday, 16:20 – 17:00

Spectral simplex method

Vladimir Protasov
Lomonosov Moscow State University

The probem of finding the maximal and minimal spectral radius of a matrix over a compact set of nonnegative matrices is notoriously hard. This is natural in view of the properties of the spectral radius: it is neither concave nor convex and may loose smoothness at points of singularity. Nevertheless, for some special matrix sets this problem is efficiently solvable. We develop an iterative optimization method for matrix sets with product structure, i.e., all rows are chosen independently from given compact sets (row uncertainty sets). If all the uncertainty sets are finite or polyhedral, the algorithm finds the matrix with maximal/minimal spectral radius within a few iterations. It is proved that the algorithm avoids cycling and terminates within finite time, which can also be estimated from above. The proofs are based on spectral properties of rank-one corrections of nonnegative matrices. Some generalizations to non-polyhedral uncertainty sets, including Euclidean balls, are derived. Some applications to mathematical economics (Leontief input-output model), spectral graph theory (optimizing the spectral radius of a graph undes some conditions on each vertex), and dynamical systems (the linear switching systems) are considered.

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Monday, 17:30 – 18:00

Non-Hermitian Hamiltonians in Quantum Mechanics

Natalia Costa
Coimbra University

We investigate spectral aspects of non selfadjoint bosonic operators. We show that the so-called equation of motion method, which is well known from the treatment of selfadjoint bosonic operators, is also useful to obtain the explicit form of the eigenvalues and eigenvectors of non selfadjoint bosonic Hamiltonians with real spectrum. We also demonstrate that these operators can be diagonalized when they are expressed in terms of pseudo-bosons, which do not behave as true bosons under the adjoint transformation, but obey the Weil-Heisenberg algebra commutation relations.

Monday, 17:30 – 18:00

On parallelization of alternating linear schemes for low-rank high-dimensional optimization

Sergey Dolgov
Max Planck Institute for Dynamics of Complex Technical Systems

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Simulation of quantum or stochastic systems may involve operator equations on functions of many variables. The number of discrete coefficients, defining such functions, grows exponentially with the number of coordinates. To make computations feasible, one has to approximate all data in a structured form. In this talk, a low-rank separation of variables in the Tensor Train (TT) format is considered. A robust approach to solve linear systems or eigenvalue problems in a low-rank format is the Alternating Linear Scheme and its extensions, such as the Density Matrix Renormalization Group. It iterates over dimensions, and reduces the large initial problem to elements of one TT factor in each step. This iteration is intrinsically sequential. Having several processors, one can iterate over different parts of a tensor format simultaneously. To pass information from one node to another, one should split the tensor format with overlaps and synchronize the shared TT factors. However, the load balancing is very difficult if the sizes of each TT factor are determined adaptively up to an accuracy threshold. Moreover, the number of iterations grows with the number of processors. Explicit synchronization of neighboring nodes [Stoudenmire and White, PRB 2013] can severely hinder the scalability. We suggest an alternative scheme, where a message is actually received half an iteration after it has been sent. This allows to accelerate the solution even in an adaptive regime.

Monday, 18:00 – 18:30

Invariant polyhedra of linear operators and the Černy conjecture

Andrey Voynov
Lomonosov Moscow State University

We consider a set of nonnegative $d \times d$-matrices (i.e. with nonnegative entries) $\mathcal{A} = \{A_1, \dots, A_k\}$. This family is called *primitive* if there exist a positive product of its elements, i.e. product with positive entries. The natural question related to the primitive sets of matrices is to estimate a sharp bound on a length of a minimal positive product of its elements. In other words, what is a minimal l (that may depend on d and k) such that for any primitive family $\mathcal{A} = \{A_1, \dots A_k\}$ of $d \times d$ nonnegative matrices, there is a product of its elements $A_{i_1} \dots A_{i_l}$ of a length l with all entries positive?

It is known that in assumption that matrices from $\mathcal{A}$ have no zero columns and no zero rows, the bound on the minimal positive product is equivalent to the bound of a minimal synchronizing word for a d-state deterministic automata. The well-known Černy conjecture (that is still open) states that it can be bounded by $(d-1)^2$. As for now, the best achieved upper bound is $O(d^3)$.

The primitive families of nonnegative matrices that have no zero columns possess the following geometrical representation. As well as primitivity of a family is defined only by combinatorial structure of its matrices, i.e. by positions of nonzero elements, one can assume that all the matrices are column-stochastic (that is sum of elements in any column is equal to 1). These matrices share a common invariant $d-1$-simplex spanned by end-points of basis vectors. It turns out that the structure of family of nonnegative matrices is closely related to the geometrical properties of this simplex and the family of linear operators acting on it. In these way we present some generalizations of the Černy conjecture in terms of convex geometry, replacing simplex with an arbitrary polyhedra.

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Monday, 18:00 – 18:30

"Inflated domain" method for the Schrödinger equation in free space

Mikhail Litsarev, Ivan Oseledets
Skolkovo Institute of Science and Technology

We present a fast low-rank method for solving the time-dependent Schrödinger equation in free space. The basic idea is that to find the solution on some interval, we introduce a larger (hence the name "inflated") domain, and start some explicit multi-step scheme. There is a decreasing (in size) sequence of such domains each of ones corresponds to the particular time step. The wrong values on the boundary start to influence the solution, but in case the initial domain is sufficiently large, we will be able to recover the solution on the initial range properly.

The interesting part is the computation of the action of the matrix exponent $e^{iH\tau}$ by some vector $\psi \in \mathbb{C}$. Since the Hamiltonian (after spatial discretization) is a Toeplitz matrix, the exponent is also a Toeplitz matrix, and the action of the operator $e^{iH\tau}$ is a convolution. We prove that the coefficients of this Toeplitz matrix decay very rapidly and the convolution can be approximated by the convolution with a finite window size.

The use of larger domains requires much more memory and computational complexity. In larger dimensions, we have to increase all of the modes, and the complexity is prohibitive. To reduce the complexity, we apply low-rank matrix and tensor methods to approximate the solution on a larger domain.

Tuesday, 09:30 – 10:10

Approximation Framework for Combined Recommendations and Price Optimization

Michael Thess
prudsys AG

We describe a novel control-theoretic framework that unifies two important tasks in modern retail applications: product recommendations and price optimization. At this, we first define a suitable state space and the metrics used. Then we focus on the approximation architecture as central part of the task. Finally, we propose an efficient policy calculation in the resulting large action space.

Tuesday, 10:10 – 10:50

Regularized Matrix Factorization for Topic Modeling of Text Collections

Konstantin Vorontsov
Moscow Institute of Physics and Technology

Topic modeling is a rapidly developing branch of statistical text analysis. Probabilistic Topic Model reveals a hidden thematic structure of a text collection and finds a compressed representation of each document in terms of its topics. Practical applications of probabilistic topic models include classification, categorization, summarization and segmentation of texts, as well as promising information retrieval paradigms of exploratory search, revealing research fronts and development trends.

From a statistical perspective, probabilistic topic model defines each topic by a multinomial distribution over words, and then describes each document with a multinomial distribution over topics. From an optimization perspective, topic modeling is an ill-posed problem of approximate stochastic matrix factorization with the Kullback-Leibler divergence.

We develop a multicriteria non-Bayesian approach named *Additive Regularization of Topic Models* (ARTM). It is based on the maximization of a weighted sum of the log-likelihood and additional regularization criteria. The optimization problem is solved via a general regularized Expectation-Maximization (EM) algorithm, which can be easily applied to any combination of regularization criteria. The combination of sparsing, smoothing, and decorrelation regularizers improves the interpretability of topics with almost no loss of the likelihood [1]. The topic selection regularizer finds a tradeoff between accuracy and complexity of the matrix factorization [2]. Multimodal regularizers describe heterogeneous collections of documents accompanied by translations, authorship, links, images, usage, space and temporal data, etc. [3].

We implemented a parallel online EM algorithm for multimodal additively regularized topic modeling in the open-source project BigARTM, available at `bigartm.org` [3]. In experiments we show how it meets multiple performance indicators for large multimodal and multilingual text collections.

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Tuesday, 10:50 – 11:30

Mathematical Problems in Wireless Communications

Vladimir Lyashev
Huawei Russian Research Center

Mobile communication has a great economic importance in today's world. Since the introduction of GSM in the early 1990s, which is still the dominant mobile communication standard, it has had an big impact on our social life (including working style, social behavior and domestic living). New systems of mobile communications are continually being introduced to live up to the great variety of applications and the desires for new services. Such development would have been impossible without the digitalization of mobile communication, whereby mathematics played an indispensable supporting role. Mathematics is foundation of information and communication theory, the pathbreaker in the development of new transmission procedures and an essential instrument in the planning and optimization of mobile networks. Probability theory and discrete mathematics are used in the development of new methods of transmission; and linear, combinatorial and stochastic optimization are used in planning mobile network and radio resource management. Despite this immense progress, essential foundational problems in wireless communication are still to be resolved. An example include a network information theory for more complex multi-user systems, a theory of self-organizing networks, and the development of new paradigms for use of the scant frequency spectrum. In all these cases it is the collaboration of telecommunications engineering and mathematics that has to deliver the essential contributions to a future networks like 5G.

Tuesday, 12:00 – 12:30

Orthogonality graphs of subsets of matrices with special properties

Bakhad Bakhadly
Lomonosov Moscow State University

Nowadays binary relations on associative rings, in particular, on the matrix algebra can be investigated with the help of graph theory if we study the so-called *relation graph* whose vertices are the elements of some set and two vertices are connected by an edge if and only if the corresponding elements are in this relation. Commuting graphs and zero-divisor graphs are the examples of relation graphs that have been studied intensively during the past 20 years. In the paper [1] the notion of graph generated by the mutual orthogonality relation for the elements of an associative ring was introduced. The present talk is devoted to the connectedness and the diameter of the orthogonality graphs of the full matrix algebra over an arbitrary field and its subsets consisting of diagonal, diagonalizable, triangularizable, nilpotent, niltriangular, and upper triangular matrices. The relation of orthogonality can be found in [3, 4, 5] where some partial orders on matrix algebra and matrix transformations which are monotone with respect to these orders are studied. Matrix orders are widely used in various fields of algebra and have applications in mathematical statistics and many other areas of mathematics [2].

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Tuesday, 12:00 – 12:30

Discretized dynamical low-rank approximation in the presence of small singular values

Hanna Walach[1], Emil Kieri[2], Christian Lubich[3]

[1] *University of Tuebingen*

[2] *Division of Scientific Computing, Department of Information Technology, Uppsala University*

[3] *Mathematical Institute, University of Tuebingen*

Low-rank approximations to large time-dependent matrices and tensors are the subject of this talk. These matrices and tensors are either given explicitly or are the unknown solutions of matrix and tensor differential equations. Based on splitting the orthogonal projection onto the tangent space of the low-rank manifold, novel time integrators for obtaining approximations by low-rank matrices and low-rank tensor trains were recently proposed. The integrators are exact when given time-dependent matrices or tensors are already of the prescribed rank. We will focus on the Lie-Trotter method, which by standard theory is first order accurate, but the usual error bounds break down when the low-rank approximation has small singular values. Here, an error analysis will be presented, that does not suffer from singular values of the low-rank solution. In cases where the exact solution is an ε-perturbation of a low-rank matrix or tensor train, the error of the projector-splitting integrator is favorably bounded in terms of ε and the stepsize,

independently of the smallness of the singular values. Such a result does not hold for any standard integrator.

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Tuesday, 12:00 – 12:30

Rectangular Maximal-Volume Concept for Cold Start Problem in Collaborative Filtering

Alexander Fonarev

Skolkovo Institute of Science and Technology

Cold start problem in Collaborative Filtering (CF) can be solved by asking new users to rate a small seed set of representative items or asking representative users to rate a new item. The question is how to build a small seed set that can give enough taste and preference information for making good recommendations. One of the most successful approaches is based on the maximal-volume concept (maxvol). It seeks the submatrix with maximal volume in a factor of Latent Factor Model (LFM) fitted to the CF data. This submatrix corresponds to the most representative users or items. But this approach have an important drawback. The submatrix should be square, and therefore the size of the seed set provided by the method is always equal to the rank of the LFM. Contrariwise, the big size of a seed set entails high ranks of factorization. This means that the factors of the LMF may be overfitted. In the current paper, we introduce a generalization of the maxvol based approach that we call rectangular maxvol. It can be applied for searching rectangular submatrices, if the size of the seed set is greater than the rank of factorization. It is also proved by experiments that the proposed method is more accurate and computational efficient than the other existing approaches.

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Tuesday, 12:30 – 13:00

Hamiltonian sets of polygonal paths in simple assembly graphs

Natasha Ostroukhova
Lomonosov Moscow State University

This talk is based on our joint work with Alexander E. Guterman.

An assembly graph is a finite connected graph such that all its vertices are rigid vertices of valency 1 or 4. A polygonal path in this graph is a path in which every pair of consecutive edges are neighbours with respect to their common vertex and no vertex is repeated. These graphs have been used for modeling genome rearrangement and homological recombination (see [1]). Polygonal paths are used for modeling rearranged DNA segments.

During my talk I am going to consider Hamiltonian sets of polygonal paths in simple assembly graphs (see [2]). An upper bound for collection of all sets of Hamiltonian polygonal paths in simply assembly graphs will be given. Also a certain type of assembly graphs for which upper bound is achieved will be presented. For assembly graphs with small number of rigid vertices it will be proved that this is the only type of graphs for which upper bound is achieved. The properties of incidence matrices of such graphs have been investigated.

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Tuesday, 12:30 – 13:00

Kolmogorov's criterion and best C-norm multivariate approximation

Sergei Goreinov
Insitute of Numerical Mathematics of the Russian Academy of Sciences

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We discuss some (not too well-known) variants of criterion of best C-norm linear approximation in multivariate case connected, in particular, with works of A. Kolmogorov, F. John, P. Gordan. An algorithm of Remez type is presented. Unlike existing ones, this algorithm reduces the problem to a standard convex analysis one of finding the Euclidean distance to convex hull of (possibly large number of) vectors in $\mathbb{C}^n$, where n is the number of degrees of freedom in the approximation in question. Some applications including low-parametric matrix approximation are discussed.

Tuesday, 12:30 – 13:00

Tensor decompositions in recommender systems - how and why it works

Evgeny Frolov, Ivan Oseledets
Skolkovo Institute of Science and Technology

The problem of product recommendation imposes ill-defined formulation, having practitioners to deal with data incompleteness and evaluation metrics inconsistencies. In this work we present a fresh view on how tensor decomposition techniques can be effectively applied to that problem.

We show that advantages of the most commonly used matrix factorization algorithms diminish when compared to simple SVD in the user-oriented evaluation metric. We present unified view on the problem of baseline prediction and propose new method for its estimation. We than generalize to the higher dimensions allowing to incorporate additional context information into the model and generate more accurate recommendations. We implement efficient sparse Tucker ALS algorithm for learning the model and higher-order folding-in technique for generating new recommendations.

We test our methods on publicly available datasets and show that tensor-based models can significantly outperform other context-aware yet matrix-based ones.

Tuesday, 13:00 – 13:30

Linearizations of Hermitian matrix polynomials preserving the sign characteristic

Susana Furtado[1], Maria Isabel Bueno[2], Froilan Dopico[3]
[1] *Universidade do Porto*
[2] *Department of Mathematics, University of California*
[3] *Department of Mathematics, Universidad Carlos III de Madrid*

The most widely used approach to solving the polynomial eigenvalue problem $P(\lambda)x = 0$ is to consider a linearization of the matrix polynomial $P(\lambda)$ and solve the corresponding linear eigenvalue problem. It is important to consider linearizations preserving whatever structure a matrix polynomial $P(\lambda)$ might possess, since such linearizations preserve the properties of the eigenvalues imposed by the structure of $P(\lambda)$.

Hermitian matrix polynomials are one of the most important classes of structured matrix polynomials arising in applications and their real eigenvalues are of great interest.

The sign characteristic of a Hermitian matrix polynomial $P(\lambda)$ with nonsingular leading coefficient is a set of signs attached to the real eigenvalues of $P(\lambda)$, which is crucial for determining the behavior of systems described by Hermitian matrix polynomials.

In this talk we present a complete characterization of all the Hermitian strong linearizations that preserve the sign characteristic of a given Hermitian matrix polynomial and identify several families of such linearizations that can be easily constructed from the coefficients of the matrix polynomial.

Tuesday, 13:00 – 13:30

Global optimization based on tensor train representation

Dmitry Zheltkov[1], Eugene Tyrtyshnikov[2]
[1] *Lomonosov Moscow State University*
[2] *Institute of Numerical Mathematics of the Russian Academy of Sciences*

Global optimization is a very important problem as it arises in many practical studies.

Most of the universal solvers for this problem (such as genetic algorithms) do not exploit the structure of an optimizing functional. Thus, on many practical tasks a specialized methods works much better. But it is difficult to construct such methods.

We suggest a new method based on tensor train representation. The method uses tensor structure of optimizing functional.

The method was applied for parameter estimation of HIV model and for molecular docking problem. Its performance on these tasks is at least as good as the best available

open source global optimization solvers, and in many cases its performance is much better than others.

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Tuesday, 13:00 – 13:30

A bi-projection method for numerical modeling of viscoplastic Bingham medium flows

Larisa Muravleva

Lomonosov Moscow State University

We propose and study a new numerical scheme to compute the isothermal and unsteady flow of an incompressible viscoplastic Bingham medium. The main difficulty, for both theoretical and numerical approaches, is due to the non-differentiability of the plastic stress tensor in areas where the deformation tensor vanishes. This is handled by introducing a projection formulation for the yield stress tensor. A new time scheme based on the incremental projection method for the Newtonian Navier-Stokes equations is proposed.The three-level splitting scheme consist of a second-order (BDF2) projection scheme for the first two steps followed by a plasticity step computing the plastic stress tensor.

A second-order cell-centered finite volume scheme on a staggered grid is applied for the spatial discretization. The scheme is assessed against previous published benchmark results for both Newtonian and Bingham flows in a two-dimensional lid-driven cavity at Reynolds number equals 1000.The proposed method is very successful at capturing both yielded and unyielded regions. Moreover, the proposed numerical scheme is able to reproduce the fundamental property of cessation in finite time of a viscoplastic medium in the absence of any energy source term in the equations.The evolution of the rigid zones in these unsteady flows is presented.

Wednesday, 09:30 – 10:10

On the Multi-Preconditioned Methods in Krylov Subspaces

Valery Ilin
Institute of Computational Mathematics and Mathematical Geophysics, Siberian Branch of the Russian Academy of Sciences

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Multi-preconditioning approaches arise in many applications with solving very large systems of linear algebraic systems of equations (SLAEs) which are obtained from the finite element or finite volume approximations of multi-dimensional boundary value problems for PDEs. Such examples can be presented by Alternating Direction Implicit methods (ADI) and Domain Decomposition Methods (DDM) with acceleration by aggregation, or coarse grid correction techniques. We consider the application of these approaches for different types of iterative processes in the Krylov subspaces, flexible generalized minimal residual method (FGMRES) and semi-conjugate direction (SCG or SCR) algorithms including. The main goal of the paper is the comparative analysis and classification of the large set of actual algorithms for important applied problems as well as the aspects of scalable parallelism of the methods in implementation at the multi-processor and multi-core computers.

Wednesday, 10:10 – 10:50

Transparent boundary conditions for anisotropic transversally heterogeneous media

Ivan Sofronov[1], Nikolai Zaitsev[2]
[1] *Schlumberger*
[2] *Keldysh IAM, RAS*

We consider the problem of generating high-accurate boundary conditions on an open face of the computational domain used for modeling elastic waves in anisotropic media. The approach of quasi analytical transparent boundary conditions (TBC) is used [1]. A specificity of the considered case consists of independence of elastic parameters in normal direction to the open boundary. This permits to drastically reduce the computational costs of evaluation of matrices in the discrete TBC operator. We describe the developed approach and demonstrate results of numerical tests of estimating the accuracy of generated boundary condition. Also some open questions of efficiency and stability of algorithms for calculating the matrices of TBC are discussed.

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Wednesday, 10:50 – 11:30

Numerical solution of problems for a space-fractional diffusion equation

Petr Vabishchevich
Nuclear Safety Institute of the Russian Academy of Sciences

Nowadays, non-local applied mathematical models based on the use of fractional derivatives in time and space are actively discussed. Many models involve both subdiffusion (fractional in time) and supper-diffusion (fractional in space) operators. Supper-diffusion problems are treated as evolutionary problems with a fractional power of an elliptic operator.

We have proposed [1] a computational algorithm for solving an equation for fractional powers of elliptic operators on the basis of a transition to a pseudo-parabolic equation. For the auxiliary Cauchy problem, the standard two-level schemes are applied. The computational algorithm is simple for practical use, robust, and applicable to solving a wide class of problems. This computational algorithm for solving equations with fractional powers of operators is promising when considering transient problems.

This work was supported by the Russian Foundation for Basic Research (projects 14-01-00785, 15-01-00026).

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Wednesday, 12:00 – 12:30

A complete solution of the permutability problem for Toeplitz and Hankel matrices

Vadim Chugunov[1], Khakim Ikramov[2]
[1] *Institute of Numerical Mathematics of the Russian Academy of Sciences*
[2] *Lomonosov Moscow State University*

The permutability problem for Toeplitz and Hankel matrices consists in characterizing matrix pairs (T, H) such that T is a Toeplitz matrix, H is a Hankel matrix, and $TH = HT$. This problem is significantly more complex than the corresponding problem for pairs of Hankel matrices and much more difficult than the problem about permutable Toeplitz matrices.

We give a sketch of the complete solution of the permutability problem for Toeplitz and Hankel matrices. A unified approach will be outlined to obtaining all the classes of commuting matrix pairs (T, H).

Wednesday, 12:00 – 12:30

1D+2D+3D Diffraction Problems: Integral Equations, Matrices, and Applications

Aleksei Tsupak, Yuri Smirnov, Mikhail Medvedik
Penza State University

In the present work, we solve numerically new integro-differential equations (IDEqs) of the vector problem of electromagnetic wave diffraction by a system of objects of various dimensions; the system consists of a solid inhomogeneous body, a perfectly conducting screen, and a wire antenna.

We reduce the original boundary value problem for Maxwell's equations to the system of IDEqs over the body domain, the screen surface, and the antenna [1]. To find approximate solutions to the system of IDEqs, we use the Galerkin method with the choice of compactly supported basis functions; we use piecewise linear basis functions [2].

The Galerkin method leads to dense matrices of a very large order (10^3–10^6). To reduce the amount of computation, we examine the structure of the matrices. Using the properties of Green's function and integro-differential operator it is possible to show that the main matrix is symmetric and multilevel block-Toeplitz.

To solve the problem of diffraction by complex obstacles, we use the subhierarchical approach [3]. According to this approach, one can use previously calculated matrix elements to solve the diffraction problem by a different scatterer instead of recalculating the matrix elements according to the Galerkin method.

In our talk, we'll consider several types of scattering obstacles; computational results will also be presented.

Acknowledgements. The research was supported by the Russian Science Foundation (project no. 14-11-00344).

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Wednesday, 12:30 – 13:00

Unitary automorphisms of the spaces of Toeplitz, Hankel, and (T+H)-matrices

Abdikozha Abdikalykov[1], Khakim Ikramov[2]

[1] *Lomonosov Moscow State University, Kazakhstan Branch*

[2] *Lomonosov Moscow State University*

The unitary similarity automorphism group $\mathrm{UAut}(K)$ of a matrix set K is the set of all unitary matrices U such that

$$\forall A \in K \to U^* A U \in K.$$

Let T_n, H_n, and TH_n be the linear spaces of all complex Toeplitz, Hankel, and Toeplitz-plus-Hankel $n \times n$ matrices, respectively. After briefly describing the groups $\mathrm{UAut}(T_n)$ and $\mathrm{UAut}(H_n)$, we outline a solution of the much more difficult problem of describing the structure of the group $\mathrm{UAut}(TH_n)$. These results have some interesting applications to eigenvalue problems for Toeplitz and Toeplitz-plus-Hankel matrices.

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Wednesday, 12:30 – 13:00

Electromagnetic Optimizations Using Heuristic Algorithms Combined With the Ordinary and Approximate Forms of the Multilevel Fast Multipole Algorithm

Özgür Ergül, Barışcan Karaosmanoğlu , Can Önol
Middle East Technical University

Electromagnetic devices, e.g., antennas, filters, waveguides, metamaterials, and photonic crystals, often need to be optimized for desired radiation, transmission, and scattering characteristics at operating frequencies [1]. Heuristic algorithms, such as the genetic algorithms and particle swarm optimization methods, enable flexible and effective design procedures by allowing for non-analytical cost functions combining multiple parameters. In addition, trials required for heuristic optimizations can be performed via computational solvers, including full-wave methods, such as the finite element method, the method of moments, and the finite-difference time-domain method. On the other hand, an optimization using a heuristic algorithm usually involves hundreds and even thousands of trials, requiring fast solvers and acceleration algorithms for performing such an optimization with a reasonable processing time.

In this study, we present an optimization mechanism involving heuristic algorithms and the multilevel fast multipole algorithm (MLFMA) [2],[3]. While there is no restriction on the type of the optimization algorithm, we use a genetic algorithm to optimize various structures in diverse application areas. Electromagnetic problems are solved iteratively using MLFMA, which performs $N \times N$ dense matrix-vector multiplications in $O(N \log N)$ time and using $O(N \log N)$ memory. MLFMA has a controllable accuracy, allowing for highly accurate solutions of complex problems with three-dimensional structures. In addition, we combine the genetic algorithm and MLFMA efficiently by facilitating setup storage strategies and lookup tables. However, despite the high efficiency and low computational complexity of MLFMA, electromagnetic solutions become bottlenecks of heuristic optimizations. Therefore, we further use the advantage of MLFMA and its controllable accuracy for effective optimizations.

Approximate forms of MLFMA were presented for constructing effective preconditioners in iterative solutions [4]. In this work, we use a similar concept for improving the optimizations, i.e., instead of a fixed accuracy, we use different MLFMA versions with different accuracies. In fact, MLFMA is so flexible that its accuracy can easily be controlled via the number of harmonics. Hence, a single MLFMA implementation can be used parametrically without dense programming efforts. Consequently, as opposed to optimizations with the method of moments, where solutions are completed a priori and cannot be manipulated during optimizations, those with MLFMA involve solutions with variable accuracy. Such a mechanism allows for less accurate but faster solutions for less critical trials (e.g., those at the beginning of an optimization) together with slower but more accurate solutions for more critical trials. While one can use alternative strategies for the dynamic control of the accuracy, optimizations of antennas show that the best results are obtained when the accuracy of MLFMA evolves as a part of the optimization process. The proposed optimizations are faster than those with the conventional MLFMA, without sacrificing the quality of the final results.

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Wednesday, 13:00 – 13:30

Lower triangular Toeplitz systems and the sequence of Bernoulli numbers

Carmine Di Fiore[1], Francesco Tudisco[2], Paolo Zellini[1]

[1] *Tor Vergata*

[2] *Department of Mathematics and Computer Science - Saarland University*

By using one of the definitions of the Bernoulli numbers, we prove that they satisfy particular lower triangular Toeplitz (ltT) systems of linear equations. In a paper Ramanujan writes down some linear equations involving Bernoulli numbers; we observe that such equations are equivalent to a ltT system. The coefficients of the ltT systems satisfied by Bernoulli numbers turn out to be all easily computable.

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Wednesday, 13:00 – 13:30

Perspectives of low rank approximation in molecular modelling

Alexander Shapeev
Skolkovo Institute of Science and Technology

Developing accurate and efficient models for interatomic interaction is crucial for molecular modelling. Typically, one of the two models is chosen: either a quantum-mechanical model which is very accurate but very computationally expensive, or an empirical model, that is very computationally efficient, but are not sufficiently accurate for qualitative predictions. A popular approach to circumvent these difficulties is to assume a sufficiently general model of interaction and "train" it (in the machine learning sense) to represent well a given quantum-mechanical model.

In my talk I will give an overview of the problem of constructing accurate and efficient interatomic interaction models and then will share my vision of the perspectives of applying the recent developments in low-rank tensor approximation to this problem.

Wednesday, 13:30 – 14:00

Structure of operator norms

Tigran Saluev
Lomonosov Moscow State University

Operator norms are a special family of norms on matrix spaces which reflect properties of matrices as bounded operators between normed spaces. Computation of operator norms has applications in various areas, including robust optimization and solving certain NP-hard problems. In this work, we focus on the question "is a given matrix norm an operator norm or not?" In order to answer it, we study properties of operator norms, their subdifferentials, dual norms and unit spheres. We provide a simple criterion for "operatorness" of a norm based on evaluating its subdifferentials.

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Wednesday, 13:30 – 14:00

Grid-based electronic structure calculations: the tensor decomposition approach

Maxim Rakhuba, Ivan Oseledets
Skolkovo Institute of Science and Technology

In the present work we focus on the development of efficient and accurate numerical techniques for solving Hartree-Fock (HF), Kohn-Sham (KS) and orbital-free density functional theory (OFDFT) equations that arise in electronic structure computations [1].

We present a fully grid-based approach for solving these equations based on low-rank approximation of three-dimensional electron orbitals [2]. Due to the low-rank structure the total complexity of the algorithm depends linearly with respect to the one-dimensional grid size. Linear complexity allows for the usage of fine grids, e.g. 8192^3. In contrast to [3] our approach is fully grid-based and, therefore, cheap and straightforward extrapolation procedure can be used.

We test the proposed approach on closed-shell atoms up to the argon, several molecules and clusters of hydrodgen atoms. We compare the total energy with highly accurate results from [4]. We also present results for OFDFT on large aluminium clusters.

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Wednesday, 15:00 – 15:40

A QTT-Accelerated Solver for Integral Equations in Complex Geometries

Denis Zorin
Courant Institute of Mathematical Sciences

I will discuss the application of QTT (quantized tensor-train) representation of matrices to acceleration of solution of integral equations on surfaces and 3D domains. We show that the low-rank structure of matrices derived from integral equations leads to a low rank QTT form, for which a highly compressed representation of an approximate inverse can be computed. For a range of problems, computational cost and storage of this representation are significantly lower than state-of-the-art methods, with sublinear (logarithmic) complexity experimentally observed. Approximate inverses in QTT format can be used as high-quality preconditioners for an iterative solver providing a flexible trade-off between performance and memory. In the case of 3D volumetric integral equations, resulting solver significantly outperforms previously-proposed methods.

Wednesday, 15:40 – 16:20

Generalized Fiedler pencils and their numerical stability

Paul Van Dooren
Catholic University Louvain

In this talk we give a new class of linearizations of a polynomial matrix

$$P(s) = P_0 + P_1 s + \ldots + P_d s^d$$

which could be called Fiedler-like linearizations of the polynomial matrix $P(s)$. The class contains the pencils defined in the work of Fiedler, but also those in the work of Antonio and Vologiannidis. The characterization of the pencils is quite straightforward and has a block structure that is much easier to describe than the derivations that were given in these earlier paper. Moreover, we show that applying the QZ algorithm to these pencils results in a backward error that can be mapped to the coefficients $P_i, i = 0 : d$, and this is a backward stable manner.

Wednesday, 16:20 – 17:00

Nonnegative rank depends on the field

Yaroslav Shitov

National Research University Higher School of Economics

In 1993, Cohen and Rothblum published the foundational paper [1], which contained a description of possible applications of nonnegative rank and an algorithm computing this function. Recall that the nonnegative rank of a matrix A over a field $\mathcal{F} \subset \mathbb{R}$ is the smallest integer k such that A is a sum of k nonnegative rank-one matrices over $\mathcal{F}$. We denote this quantity by $\mathrm{rank}_+(A, \mathcal{F})$.

The algorithm by Cohen and Rothblum was based on quantifier elimination, and they did not know whether it is applicable to compute nonnegative ranks over fields which are not real closed. Cohen and Rothblum posed the question: How sensitive is the problem to the ordered field over which it is asked?

Problem. Do there exist a subfield $\mathcal{F} \subset \mathbb{R}$ and a nonnegative matrix $A \in \mathcal{F}^{m\times n}$ such that $\mathrm{rank}_+(A, \mathcal{F}) \neq \mathrm{rank}_+(A, \mathbb{R})$?

Now the nonnegative factorization is an important tool in applied mathematics. The complexity of determining nonnegative rank is much better understood now, and faster algorithms have been developed. However, the problem mentioned above remained open. I am going to talk about the positive solution of this problem, which I give in [2]. One particular implication of this result is that algorithms based on quantifier elimination cannot help to compute nonnegative factorizations over arbitrary fields. One can ask: Is there an algorithm computing $\mathrm{rank}_+(A, \mathbb{Q})$, given a rational nonnegative matrix A? A more specific question asked by Cohen and Rothblum [1] is also open: Is the equality $\mathrm{rank}_+(A, \mathbb{R}) = \mathrm{rank}_+(A, \mathbb{Q})$ always true?

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Wednesday, 17:30 – 18:00

Deterministic bounds on the restricted isometry in compressed sensing matrices

Igor Kaporin
Federal Research Center

The construction and analysis of compressed sensing (CS) matrices is the key point in the CS research [1]. We consider the standard formulation of the restricted isometry property (RIP) required for a rectangular $m \times n$ CS matrix A with $m < n$. Here, one needs to estimate the linear independence for any subset of $k < m$ columns which form an $m \times k$ submatrix $[a_{j(1)}, \dots, a_{j(k)}] = A_J$ of A. The standard RIP condition implies

$$\mathrm{cond}(A_J^T A_J) \le \frac{1+\delta_k}{1-\delta_k},$$

where $0 < \delta_k < 1$ is the so called RIP constant of the order k. For the latter quantity, we present a lower bound which holds for any CS matrix. Namely, assuming $AA^T = \frac{n}{m} I_m$ and $(A^T A)_{ii} = 1$, we prove (under the simplifying assumption $\delta_k < 0.847$) that there always exist such index subset $J = \{j(1), \dots, j(k)\}$ that

$$\frac{1}{m} < \frac{2\delta_k^2}{k-1} + \frac{1}{n}.$$

It must be stressed that the result is completely deterministic. The proof is based on a generalization of the Binet-Cauchy determinant identity and inequalities presented in [2]. Comparing the obtained result with the known sufficient conditions on δ_k and k (see, e.g., [1]) used to guarantee the performance for varous CS solvers, one can figure out the resulting lower bounds on the undersampling ratio m/n and/or upper bounds on sparsity (i.e., a multiple of k/n).

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Wednesday, 17:30 – 18:00

Block LU preconditioner for some integral equation

Stanislav Stavtsev, Vadim Chugunov
Institute of Numerical Mathematics of the Russian Academy of Sciences

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The radiolocation problem is often solved using integral equations. The simplest model when the given object is a perfectly conducting body is characterized by an integral equation of the electric field. The solution of a system with a dense matrix of size $10^4 - 10^6$ requires computing systems with a large RAM or special-purpose methods for compressing the dense matrix. We have used mosaic-skeleton approximations.

To compute of the radar cross section (RCS) the system with several right-hand sides is solved by a modification of GMRES that uses basis vectors of the projective subspace for all right-hand sides of the system. This system could be solved for each right-hand side or the GMRES restarts could be used since the system solution time increases so greatly that it becomes impossible to solve the integral equation for $k > 10$. But modified GMRES solver needs large memory.

We propose to build a preconditioner, using low-rank structure of matrix. With new preconditioner the frequency of solved electro-magnetic problem is 4 times larger.

Wednesday, 17:30 – 18:00

On numerical range and numerical radius for some integral operators

Abdelaziz Mennouni
Department of mathematics, LTM, University of Batna

We present the cartesian decomposition of integral operator. Several new inequalities concerning the numerical radius for some integral operators on a Hilbert space are considered. We also discuss the numerical ranges of some integral operators.

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The characterization of operators preserving scrambling index for matrices over semirings

Artem Maksaev
Lomonosov Moscow State University

This talk is based on our joint work with Alexander E. Guterman.

In 2009 Akelbek and Kirkland [1] defined the scrambling index of a nonnegative primitive matrix A, denoted by $k(A)$, which is the smallest positive integer k such that any two rows of A^k have at least one positive element in coincident position. This concept proved to be important for applications in the theory of communication. Moreover, scrambling index is closely related with well-known Dobrushin or delta coefficient, denoted by $\tau_1(\cdot)$, which has applications, in particular, to the theory of Markov chains (see [2], Chapter 4, [3], Chapter 2, and references therein).

The aim of this paper is a complete characterization of surjective linear operators acting on the matrix space over an antinegative semiring with 1 and without zero divisors, which preserve scrambling index. Continuing the investigations of linear maps acting on matrices over semirings, for example, see [4], the following result is obtained.

Theorem. Let S be an antinegative semiring with unity 1 and without zero divisors. Let $T\colon M_n(S) \to M_n(S)$ $(n \geqslant 3)$ be a surjective linear operator which preserves scrambling index, then there is a permutation matrix P and a matrix $B = (b_{ij}) \in M_n(S)$, each b_{ij} has a left inverse, such that $T(A) = P^T(A \circ B)P$ for all matrices $A \in M_n(S)$, here $A \circ B$ denotes the Hadamard (elementwise) product.

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Wednesday, 18:00 – 18:30

Rectangular submatrices of maximum volume and its applications.

Alexander Mikhalev
Skolkovo Institute of Science and Technology

Submatrices with certain extreme properties arise in many applications. Common technique is to choose volume of a matrix as such extreme property. The volume of a square matrix is defined as an absolute value of its determinant. The exact computation of the submatrix of maximal volume is an NP-hard problem. However, in many applications a submatrix of sufficiently large volume is enough. Previous approaches specialize on finding only square submatrices. We extend the volume concept to the case of rectangular matrices, generalize well-known results for the square case, derive theoretical upper bounds on coefficients and propose new volume maximization algorithm. Proposed method is tested and compared with *maxvol* [1] algorithm on three promising applications of such submatrices: recommender systems, finding maximal elements in low-rank matrices and preconditioning of overdetermined linear systems.

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Wednesday, 18:00 – 18:30

Parallel explicit/implicit computing of multidimensional TiC combustion synthesis using openmp

Asdin Aoufi[1], Gilles Damamme[2]
[1] *EMSE*
[2] *CEA*

Introduction

Our work is related to the numerical simulation of **SHS** (Self-propagating High temperature Synthesis) process that is used to synthesize ceramic materials. The modeling of the process is based upon the coupling between a non-linear reaction-diffusion equation and a differential equation.

$$
\begin{aligned}
\rho C_p(T,\xi)\frac{\partial T}{\partial t} &= \nabla.\left(\lambda(T,\xi)\nabla T\right) + \Delta H k(T)\left(1-\xi\right).\\
-\lambda(T,\xi)\frac{\partial T}{\partial n} &= \epsilon\sigma\left(T - T_\infty^4\right).\\
\frac{d\xi}{dt} &= k(T)\left(1-\xi\right).
\end{aligned}
$$

Mathematical modelling

a priori energy estimates are established when the thermophysical properties of the material are either assumed constant or temperature and conversion rate depend. A continuous maximum principle for both the conversion rate $0 \leq \xi$ and the temperature T is proved.

Explicit finite-volume discretization

An explicit finite-volume scheme is written for one-dimensional (slab/cylindrical/spherical) geometry, two-dimensional (cylindrical, cartesian) geometry and three-dimensional cartesian geometry on a structured mesh. The stability condition over the time-step Δt is established for the cases where the thermophysical properties of the materials are either assumed constant or temperature and conversion rate depend. A discrete maximum principle for both the temperature and the conversion rate is proved.

Implicit finite-volume discretization and iterative matrix solvers

We establish that the sparse tridiagonal (for the 1d case) /sparse pentadiagonal (for the 2d case)/sparse heptadiagonal (for the 3d case) arising from the backward Euler time-implicit finite-volume discretisation is an unsymmetric strictly dominant $M-$matrix inducing a discrete maximum principle. The linear solvers that are compared to each others in this study are standard SSOR, Bicgstab and IBicgstab.

Parallel computing

All multidimensional arrays are converted into one-dimensional array to avoid pointer aliasing problems. For the explicit case, when the thermophysical properties are not constant, a reduction construct is used to compute the time-step imposed by the cfl condition. For the implicit case, specific storage of the sparse matrix coefficients is organized to ensure locality in cache-based hierarchies microprocessors and efficiency of the matrix-vector product. A comparison of the efficiency of both gnu and intel compiler is done. The autoparallelization of the computational kernels by the intel compiler is compared to manualy inserted openmp directives in the numerical software. We report numerical results concerning the elapsed time of both explicit and implicit scheme when the thermo-physical parameters are assumed constant or are temperature and conversion rate dependant.

Numerical simulation results

We investigate the contribution of the furnace's temperature and the expression of the thermo-physical parameters to the combustion front propagation inside the material and to the induction time.

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Thursday, 09:30 – 10:10

Current Problems in Matrix Population Models: Expanding Classics and New Discoveries

Dmitrii O. Logofet

Laboratory of Mathematical Ecology, Institute of Atmospheric Physics, Russian Acad.Sci.

Matrix population models describe the time dynamics of a single-species population that is structured (into a finite number of individual groups) by an observable attribute of the individuals: age, size, ontogenetic stage, etc, called *stage* in a generalized sense. The ensuing matrix, L, of vital rates, as the matrix of a system of (linear) difference equations $x(t+1) = Lx(t)$, is square, nonnegative, irreducible/reducible, and the classical Perron–Frobenius theorem is well known to substantiate the *Perron root*, or the dominant eigenvalue $\lambda_1(L) = \rho(L)$ as the asymptotic rate of population growth, the quantitative measure of adaptation, thus a popular tool of comparative demography. It is of practical importance to know whether $\lambda_1(L)$ is greater, equal or less than 1, and when L is not certain in all its elements, it is of practical merit to have an *indicator* function, $R(L)$, that is always located on the same side of 1 as is $\lambda_1(L)$. Matrix $L = T + F$ represents traditionally the sum if its transition and fertility parts, and the *net reproductive rate* $R_0(L) = \rho(F[I–T]^{-1})$ is well known to possess the indicator ability and widely used in the literature. Another, algebraically simpler, function $R_1(L) = 1–\det(I–L)$ is an indicator too if the second positive eigenvalue, $\lambda_2(L)$ (when it exists), is less than 1 (Logofet, 2013). In the general case confided only to the rank-one F, this principal location of $\lambda_2(L)$ was recently inferred from the theory of rank-one corrections in nonnegative matrices (Protasov, Logofet, 2014).The main theorem of this theory established the spectral radius of the corrected matrix as its sole eigenvalue that can be greater than the spectral radius prior to correction. In the "correction", $L = T + F$, of T by a rank-one F, matrix T is substochastic due to the sense of its elements, hence $\rho(T) < 1$, and it follows that the positive $\lambda_2(L)$ cannot be greater than $\rho(T) < 1$. A recent botanic case study has however revealed situations where matrix F has rank 2 or even 3 (Logofet et al., 2015), yet $R_1(L)$ still possessing the indicator property in those practical cases. To give this observation a general explanation in terms of matrix patterns is another challenge in the mathematics of matrix population models. The same case study has also discovered a new phenomenon in the ontogenesis of a perennial plant species and a novel situation in matrix models where the famous $\lambda_1(L)$ loses its accuracy in comparative demography and needs a correction. Any irregular schedule of transitions between stages induces a nontrivial problem to infer the age-specific parameters of the population from its stage-structured model, and there is a technique of virtual absorbing Markov chains (VAMC) that solves the problem via the fundamental matrices of the chains (Caswell, 2001). However, in a nonautonomous model $L(t)$, $t = 0, 1, \ldots, k$, the VAMC technique results in a variety of k time-specific estimates, and another nontrivial problem arises: how to calculate the geometric mean for several nonnegative (asymmetric and non-commutative) matrices of a given pattern. The problem still remains unsolved, but the stochastic transition matrices, these intermediates of the VAMC method, give certain grounds to anticipate a future progress in the averaging problem for their substochastic prototypes, $T(t)$.

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Thursday, 10:10 – 10:50

QR algorithm, blurred shifts and the computation of the Jordan structure of an eigenvalue

Nicola Mastronardi[1], Paul Van Dooren[2]

[1] *Istituto per le Applicazioni del Calcolo, CNR*

[2] *Department of Mathematical Engineering, Catholic University of Louvain*

In this paper we revisit the problem of finding an orthogonal similarity transformation that puts an $n \times n$ matrix A in a block upper-triangular form that reveals its Jordan structure at a particular eigenvalue α. The obtained form in fact reveals the dimensions of the null spaces of $(A - \alpha I)^i$ at that eigenvalue via the sizes of the leading diagonal blocks, and from this the Jordan structure at α is then easily recovered. The method starts from a Hessenberg form that already reveals several properties of the Jordan structure of A. It then updates the Hessenberg form in an efficient way to transform it to a block-triangular form in $O(mn^2)$ floating point operations, where m is the total multiplicity of the eigenvalue. The method only uses orthogonal transformations and is backward stable. We illustrate the method with a number of numerical examples.

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Thursday, 10:50 – 11:30

On descent algorithms on low-rank matrix varieties

Andre Uschmajew
University of Bonn

A way to address matrix optimization problems with low-rank constraints are Riemannian optimization algorithms on the smooth manifold of, say, rank-k matrices. If sufficiently gradient-related, the corresponding line-search methods can be seen as discrete gradient flows (or dynamical systems). Similar iterations arise in the context of geometric integrators for dynamical low-rank approximation which attracted recent attention. If not given in advance, a practical problem can be the choice of a suitable rank k. But even if the "correct" target rank is chosen, the rigorous convergence analysis is hampered by the fact that the manifold of rank-k matrices itself is not closed.

We show how both problems can be addressed by formally considering gradient iterations on the variety of matrices of rank at most k, in which the tangential gradient projections in rank-deficient points involve the tangent cones instead of tangent spaces. These tangent cones turn out to have a simple explicit description. Convergence results for specific step-size rules can then be obtained via Łojasiewicz gradient inequality, allowing rank-deficient limit points. Conversely, with this approach, (locally) optimal points with rank at most k can be embedded as starting guess into a variety of higher rank, e.g., $k+1$. In this way, efficient rank-increasing heuristics can be designed. Popular strategies realize this embedding by adding low-rank approximations of the unconstrained antigradient to the current iterate. This heuristic can now be justified from the explicit form of the tangent cone again.

The talk is based on joint work with Reinhold Schneider (TU Berlin) and Bart Vandereycken (University of Geneva).

Thursday, 11:30 – 12:10

Classification problems for systems of forms and linear mappings

Vladimir V. Sergeichuk
Institute of Mathematics

I will say about the method of reducing the problem of classifying systems of bilinear or sesquilinear forms and linear mappings to the problem of classifying systems of linear mappings that was developed in [1] (see also [2–5]). I will give applications of this method.

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Friday, 09:30 – 10:10

Toeplitz matrices in multidimensional inverse problems

Sergey Kabanikhin[1], Nikita Novikov [2], Ivan Oseledets[3], Maxim Shishlenin [4]

[1] *Institute of Computational Mathematics and Mathematical Geophysics SB RAS*
[2] *Novosibirsk State Unversity*
[3] *Skolkovo Institute of Science and Technology*
[4] *Sobolev Institute of Mathematics SB RAS*

The coefficient inverse problem for the acoustic equation is considered. The method for reconstructing the density is proposed based on the N-approximation by the finite system of one-dimensional problems and two-dimensional M.G. Krein approach [2, 3]. Two-dimensional analog of the M.G. Krein approach is applied to reduce the nonlinear inverse problem to a family of linear integral equations. Fast algorithm for solving the relevant linear system, based on using the block-Toeplitz structure of the matrix, is considered [1]. It is shown that the structure of the algorithm combined with the consideration of the M.G. Krein equation allows to solve the whole family of the integral equations by solving only one linear system. Results of numerical calculations are presented.

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Friday, 10:10 – 10:50

Eigenvalue algorithms for Hermitian quasiseparable of any order matrices

Yuli Eidelman, Iulian Haimovici

Tel-Aviv University

We present and compare various fast algorithms for matrices with quasiseparable representations. It is demonstrated how the algorithms developed for order one quasiseparable matrices may be extended for quasiseparable representations with higher orders.

Friday, 10:50 – 11:30

Subdiagonal rational approximation of the function $z^{-1/2}$ on the union of a positive and a negative real line segment

Leonid Knizhnerman[1], Vladimir Druskin[2], Stefan Güttel[3]

[1] *Central Geophysical Expedition*

[2] *Schlumberger-Doll Research*

[3] *The University of Manchester, School of Mathematics*

When constructing absorbing boundary conditions for solving discretized hyperbolic PDEs, one needs to know good uniform $[n-1,n]$ rational approximants to the function $z^{-1/2}$ defined with the slit along the negative imaginary semiaxis. The compact set K, on which the chosen branch of the function $z^{-1/2}$ is approximated, equals the union of two real line segments separated by the imaginary axis: $K = [a_1, b_1] \cup [a_2, b_2]$, $a_1 < b_1 <$

$0 < a_2 < b_2$. We present a constructive solution for an approximation problem almost minimizing the relative error and derive tight upper and lower bounds for it. Relative rational approximation problems are also considered.

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Friday, 12:00 – 12:30

Monotone maps on matrix and operator algebras with respect to sharp partial order

Mikhail Efimov
Lomonosov Moscow State University

There is a lot of partial orders that can be defined on matrix space. Some of these orders are useful in matrix investigations and applications such as quantum mechanics, mathematical statistics, calculus mathematics. We concentrate our attention on sharp parial order (see [1]).

Let $M_n(\mathbb{F})$ be a linear space of $n \times n$-matrices with entries from an arbitrary field $\mathbb{F}$. We say that $A^\sharp$ is *the group inverse matrix* for the matrix A if it satisfies the following conditions: $AA^\sharp A = A$, $A^\sharp AA^\sharp = A^\sharp$, $AA^\sharp = A^\sharp A$. Then for two matrices $A, B \in M_n(\mathbb{F})$ we write $A \overset{\sharp}{\leqslant} B$, if and only if $A = B$ or there exists $A^\sharp$ and $AA^\sharp = BA^\sharp = A^\sharp B$. This relation is reflexive, antisymmetric and transitive. So, it is called *sharp order*.

We investigate monotone maps with respect to $\overset{\sharp}{\leqslant}$-order. Full characterization of such monotone maps can be obtained in different cases. For example, additive monotone maps are characterized in [2].

Theorem. *Let $\mathbb{F}$ be an arbitrary field with $|\mathbb{F}| \geqslant 3$, $n \geqslant 2$ an integer. Suppose an additive map $T\colon M_n(\mathbb{F}) \to M_n(\mathbb{F})$ is monotone with respect to $\overset{\sharp}{\leqslant}$–order; then T has one of the two following forms:*
1) $T(X) = \alpha P^{-1} X^\varphi P$ for all $X \in M_n(\mathbb{F})$;
2) $T(X) = \alpha P^{-1} (X^\varphi)^t P$ for all $X \in M_n(\mathbb{F})$;
(here $\alpha \in \mathbb{F}$, $P \in GL_n(\mathbb{F})$, $\varphi\colon \mathbb{F} \to \mathbb{F}$ is an injective endomorphism of $\mathbb{F}$).

Non-additive maps which are monotone with respect to $\overset{\sharp}{\leqslant}$-order are also investigated. Monotone maps on set of diagonalizable matrices are characterized in [3]. Furthermore, we can characterize continuous monotone maps on complex matrices. In particular, all these maps should be additive.

In addition, the analog of sharp order for set $B(H)$ of linear bounded operators in Hilbert space can be defined. We obtain characterization of additive bijective monotone maps on $B(H)$. This result may be useful in view of its applications to quantum mechanics.

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Friday, 12:00 – 12:30

Fast block low-rank direct solver for sparse matrices

Daria Sushnikova

Institute of Numerical Mathematics Russian Academy of Sciences

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We propose fast direct sparse solver, based on the idea that inversion of sparse matrix A has $\mathcal{H}^2$ structure [5, 1].

Our goal is to build approximate sparse factorization. Using the assumption that inverse matrix has $\mathcal{H}^2$ structure we reduce fill-in in factorization by approximating far block rows. As a result of the algorithm we obtain

$$A = QP^T LL^T PQ^T,$$

where Q is FMM like matrix [3, 4], L is triangular matrix, P is permutation matrix.

Similar idea of using block low-rank structure of inverse matrix is presented in $\mathcal{H}$-lib package [6], algorithm has $O(N)$ complicity, but it has a big constant. Idea of block rows approximation is presented in paper [2], but authors approximate whole of-diagonal part of block row, that leads to big computational complexity. We propose $O(N)$ complexity algorithm with small constant that is robust and easy in implementation.

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Friday, 12:00 – 12:30

A Low-Rank Optimization Algorithm based on the Projector Splitting Integrator

Sebastian Wolf, Benjamin Huber, Reinhold Schneider
Technische Universität Berlin

The hierarchical Tucker (HT) format [1] and its powerful special case the Tensor Train (TT) format [2] have proven to be very efficient tools for the approximation of high dimensional tensor problems. A particular promising approach is the view of the set of tensors with fixed TT-rank as a Riemannian manifold [3]. This allows to efficiently solve a large class of optimization problems on this set by Riemannian optimization.

In the present talk we consider a variant of the recently introduced projector splitting integrator [4] to obtain algorithms for optimization problems on the TT-manifold. As an example we consider the application to the popular tensor completion problem.

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Friday, 12:30 – 13:00

Extremal matrix centralizers and their applications

Alexander Guterman
Lomonosov Moscow State University

This talk is based on our recent joint works with G. Dolinar, B. Kuzma and P. Oblak [1, 2, 3].

For $A \in M_n(\mathbb{F})$ its *centralizer* $\mathcal{C}(A) = \{X \in M_n(\mathbb{F}) | \; AX = XA\}$ is the set of all matrices commuting with A. For a set $S \subseteq M_n(\mathbb{F})$ its centralizer $\mathcal{C}(S) = \{X \in M_n(\mathbb{F}) | \; AX = XA$ for every $A \in S\}$ is the intersection of centralizers of all its elements. The centralizer is important and useful notion both in fundamental and applied sciences.

A non-scalar matrix A is *minimal* if for every $X \in M_n(\mathbb{F})$ with $\mathcal{C}(A) \supseteq \mathcal{C}(X)$ it follows that $\mathcal{C}(A) = \mathcal{C}(X)$. A non-scalar matrix A is *maximal* if for every non-scalar $X \in M_n(\mathbb{F})$ with $\mathcal{C}(A) \subseteq \mathcal{C}(X)$ it follows that $\mathcal{C}(A) = \mathcal{C}(X)$.

We investigate and characterize minimal and maximal matrices over arbitrary fields.

Our results are illustrated by two applications. The first one is in the theory of commuting graphs of matrix rings. The second one is the characterization of commutativity preserving maps on matrices, which belongs to the theory of maps preserving matrix invariants.

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Friday, 12:30 – 13:00

Adaptive matrix algebras in unconstrained optimization

Stefano Cipolla, Carmine Di Fiore
Tor Vergata

In this communication we will introduce some recent techniques which involve structured matrix spaces in the reduction of time and space complexity of BFGS-type minimization algorithms [1],[2]. The first part of the communication deals with the main theoretical instruments used to achieve this complexity reduction, essentially related with the projection of a matrix onto a given subspace. The projected matrix inherits from the original one many relevant spectral properties which reveal to be crucial for the right development of the theoretical machinery. In the second part of the communication, some general results for the global convergence of algorithms for unconstrained optimization based on a BFGS-type Hessian approximation scheme are introduced. Then it is shown how the constructibility of convergent algorithms suitable for large scale problems can be tackled using the properties of the projection operator. Attention is focused on the most recent development of such techniques [2], where the structured space chosen for the projection is adaptively selected at each step in order to obtain an accurate approximation of the search directions produced by the BFGS scheme, with the aim of maximizing the gain of efficiency.

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Friday, 12:30 – 13:00

Fast low-rank solution of the Poisson equation based on cross approximation

Ekaterina Muravleva[1], Ivan Oseledets[2]

[1] *Max Planck Institute for Mathematics in the Sciences*

[2] *Skolkovo Institute of Science and Technology*

We consider the problem of computing approximate solution of Poisson equation in the low-parametric tensor formats. We propose a new algorithm to compute the solution based on the cross approximation algorithm in the frequency space, and it has better complexity with respect to ranks in comparison with standard algorithms, which are based on the exponential sums approximation. To illustrate the effectiveness of our solver, we incorporate into a Uzawa solver for the Stokes problem on semi-staggered grid as a subsolver. The resulting solver outperforms the standard method for $n \geq 256$.

Friday, 13:00 – 13:30

Fully indecomposable non-convertible (0,1)-matrices

Mikhail Budrevich[1], Gregor Dolinar[2], Bojan Kuzma[3], Alexander Guterman[1]

[1] *Lomonosov Moscow State University*

[2] *University of Ljubljana*

[3] *University of Primorska*

The problem of convertibility of a given (0,1)-matrix is known as Polya convertibility problem. It is related to some problems in graph theory such as: if a (0,1)-matrix is sign nonsingular or does a bipartite graph have a Pfaffian orientation? A matrix A is called sign convertible if we can multiply some of its elements by -1 in such a way that for obtained matrix A' the equality $\operatorname{per}(A) = \det(A')$ holds. It is known that any nonconvertible (0,1)-matrix contains at least $n+6$ non-zero entries.

In this talk we consider sign convertibility problem for fully indecomposable (0,1)-matrices. It is proved that any fully indecomposable nonconvertible (0,1)-matrix contains at least $2n+3$ non-zero entries. Moreover, the characterization of fully indecomposable non-convertible (0,1)-matrices with exactly $2n+3$ non-zero entries is obtained.

Friday, 13:00 – 13:30

Regularization preconditioners for frame-based image deblurring with reduced boundary artifacts

Davide Bianchi[1], Marco Donatelli[1], Yuantao Cai[2], Ting-Zhu Huang[2]

[1] *Dipartimento di Scienza e Alta Tecnologia, Università dell'Insubria*

[2] *School of Mathematical Sciences, University of Electronic Science and Technology of China*

Image deblurring is the process of reconstructing an approximation of an image from blurred and noisy measurements. By assuming that the point spread function (PSF) is spatially-invariant, the observed image $g(x, y)$ is related to the true image $f(x, y)$ via the integral equation

$$g(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(x-x', y-y') f(x', y')\, dx'\, dy' + \eta(x,y), \qquad (x,y) \in \Omega \subset \mathbb{R}^2, \quad (5)$$

where $\eta(x, y)$ is the noise.

By collocation of the previous integral equation on a uniform grid, we obtain the grayscale images of the observed image, of the true image, and of the PSF, denoted by $\boldsymbol{G}$, $\boldsymbol{F}$, and $\boldsymbol{H}$, respectively. Since collected images are available only in a finite region, the field of view (FOV), the measured intensities near the boundary are affected by data outside the FOV. Given an $n \times n$ observed image $\boldsymbol{G}$, and a $p \times p$ PSF with $p \leq n$, then $\boldsymbol{F}$ is $m \times m$ with $m = n + p - 1$. Denoting by $\boldsymbol{g}$ and $\boldsymbol{f}$ the stack ordered vectors corresponding to $\boldsymbol{G}$ and $\boldsymbol{F}$, the discretization of (5) leads to the under-determined linear system

$$\boldsymbol{g} = A\boldsymbol{f} + \boldsymbol{\eta}, \quad (6)$$

where the matrix A is of size $n^2 \times m^2$ or square $n^2 \times n^2$ when imposing proper Boundary Conditions (BCs).

Due to the ill-posedness of (5), A is severely ill-conditioned and may be singular and therefore a good approximation of $\boldsymbol{f}$ cannot be obtained from the algebraic solution (e.g., the least-square solution) of (6), but regularization methods are required. The basic idea of regularization is to replace the original ill-conditioned problem with a nearby well-conditioned problem, whose solution approximates the true solution.

Thresholding iterative methods are recently successfully applied to image deblurring problems. In this talk, we investigate the modified linearized Bregman algorithm (MLBA) [1] used in image deblurring problems, with a proper treatment of the boundary artifacts. The fast convergence of the MLBA depends on a regularizing preconditioner that could be computationally expensive and hence it is usually chosen as a block circulant circulant block (BCCB) matrix, diagonalized by discrete Fourier transform. We show that the standard approach based on the BCCB preconditioner may provide low quality restored images and we propose different preconditioning strategies, that improve the quality of the restoration and save some computational cost at the same time.

In particular, motivated by a recent nonstationary preconditioned iteration [4, 5], we propose a new algorithm that combines such method with the MLBA. We prove that it is a regularizing and convergent method.

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Friday, 13:30 – 14:00

On rank-one perturbations of stable matrices

Volha Kushel

Shanghai Jiao Tong University

In this talk, we focuse on the following question: when do rank-one perturbations of a matrix preserve its positive stability? We study the class of $\boldsymbol{P}^2$-matrices and its subclasses that contain structured matrices. We also study the link of this problem to the problem of characterizing the class of $\boldsymbol{D}$-stable matrices and the problem of positive solvability of linear systems.

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Friday, 13:30 – 14:00

Probabilistic graphical models: a tensorial perspective

Anton Rodomanov[1], Alexander Novikov[2], Anton Osokin[3], Dmitry Vetrov[4]

[1] *Lomonosov Moscow State University*

[2] *Skolkovo Institute of Science and Technology*

[3] *SIERRA, INRIA*

[4] *National Research University Higher School of Economics and Skolkovo Institute of Science and Technology*

Probabilistic graphical models have become a popular tool for many applications in computer vision, machine learning, social networking etc. One of their advantages is the ability to define the joint probability distribution of thousands of variables in a compact form (e.g. a distribution over the set of all possible labellings in an image segmentation problem). We propose a new framework for dealing with probabilistic graphical models. The central idea in this framework is to treat the joint probability distribution of a probabilistic graphical model as a tensor. Our approach relies on the recently proposed Tensor Train format [1] which is compact and allows for an efficient application of linear algebra operations. We present a way to convert the energy tensor to the Tensor Train format and show how one can exploit the properties of this format to address the tasks of estimating the normalisation constant and finding the most probable configuration. We provide theoretical guarantees on the accuracy of the proposed algorithm for estimating the normalisation constant and compare our methods against several state-of-the-art algorithms.

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Friday, 13:30 – 14:00

Tensor train approximation in active subspace variables with application to parametric partial differential equations.

Andrei Chertkov[1], Ivan Oseledets[2]

[1] *Skolkovo Institute of Science and Technology*

[2] *Skolkovo institute of science and technology (Skoltech)*

We propose a method for solving parametric partial differential equations. The dimension of the equation in the parameter space is reduced by switching to the active subspace [1] - the low-dimensional subspace that is constructed according to directions of the strongest variability of the parameters. Desirable functional of the solution is interpolated in the active subspace as a function of parameters with the use of multi-dimensional cross-approximation [2] for acceleration of the training values calculation and tensor train format [3] for the compressed representation of the arising tensors and fast operating under them.

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Friday, 15:00 – 15:40

Majorization inequalities for valuations of eigenvalues using tropical algebra

Marianne Akian

INRIA and CMAP, Ecole Polytechnique

We consider a matrix with entries over the field of Puiseux series, equipped with its non-archimedean valuation (the leading exponent). We establish majorization inequalities relating the sequence of the valuations of the eigenvalues of a matrix with the tropical eigenvalues of its valuation matrix (the latter is obtained by taking the valuation entry-wise). We also show that, generically in the leading coefficients of the Puiseux series, the precise asymptotics of eigenvalues, eigenvectors and condition numbers can be determined. For this, we apply diagonal scalings constructed from the dual variables of a parametric optimal assignment constructed from the valuation matrix.

Next, we establish an archimedean analogue of the above inequalities, which applies to matrix polynomials with coefficients in the field of complex numbers, equipped with the modulus as its valuation.

This talk covers joint works with Ravindra Bapat, Stéphane Gaubert, Andrea Marchesini, and Francoise Tisseur.

Friday, 15:40 – 16:20

Error Estimation for Ill-Posed Problems

Anatoly Yagola

Lomonosov Moscow State University

It is very well known that in a general case it is impossible to provide error estimation for ill-posed problems without an additional a priori information about the unknown solution. In this presentation two types of a priori information are considered:

1) the solution is sourcewise represented;

2) the sollution is an element of a given compact set.

I thank the RFBR for a financial support (grants 14-01-00182, 14-01-91151-NSFC).

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Friday, 16:20 – 17:00

Spectra of Random Matrices and Riemann-Hilbert Problems for Matrix Valued Analytic Functions

Alexander Aptekarev
Keldysh Institute of Applied Mathematics of the Russian Academy of Sciences

At the end of XX century Deift and Zhou have put forward a new technique of asymptotical analysis, namely steepest descent method for matrix valued oscillatory Riemann-Hilbert problems. Many important problems in various fields of mathematics (probability, mathematical physics, approximation theory and others) were solved by this powerful method. In our talk we discuss some of these applications and the main features of the matrix Riemann-Hilbert problem method. The main attention will be paid to the distributions of the scaled eigenvalues of the enlarging size random matrices.

Friday, 17:30 – 18:00

Parallel implementation of the fast method solving the kinetic equations of aggregation and fragmentation

Sergey Matveev
Lomonosov Moscow State University, Skoltech

We present a parallel version of the fast algorithm solving systems of kinetic equations of aggregation and fragmentation processes[1]. This fast method was proposed in [2] and it is based on application of low-rank approximations [3] of the kernel coefficients and the fast algorithms of linear algebra[4] to evaluation of particles collision operator.

In this work we present a direct parallel implementation of the same algorithm. We evaluate the low-rank representations of the kernel coefficients using the parallel implementation of the matrix cross interpolation algorithm[5] and formulate our algorithms for the fast linear algebra operations. The efficiency and scalability of the parallel scheme are demonstrated on the concrete problems of aggregation and fragmentation. The oscillatory solutions of the Cauchy problems for the considered systems are found using the proposed parallel algorithm. The presented parallel algorithm can be easily adopted to fast numerical methos solving continuous Smoluchowski equation[6].

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Friday, 17:30 – 18:00

Criterion of total positivity of generalized Hurwitz matrices

Mikhail Tyaglov[1], Olga Kushel[1], Olga Holtz[2], Sergei Khrushchev[3]
[1] *Shanghai Jiao Tong University*
[2] *UC Berkeley*
[3] *Kazakh-British University*

For a given set of real numbers a_0, a_1, ..., a_n, and an integer M, $2 \leqslant M \leqslant n$, the following infinite matrix

$$H_M = \begin{pmatrix} a_{M-1} & a_{2M-1} & a_{3M-1} & \cdots \\ a_{M-2} & a_{2M-2} & a_{3M-2} & \cdots \\ \vdots & \vdots & \vdots & \\ a_0 & a_M & a_{2M} & \cdots \\ 0 & a_{M-1} & a_{2M-1} & \cdots \\ 0 & a_{M-2} & a_{2M-2} & \cdots \\ \vdots & \vdots & \vdots & \\ 0 & a_0 & a_M & \cdots \\ 0 & 0 & a_{M-1} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

is called generalized Hurwitz matrix, and for $M = 2$ the matrix H_2 is a standard *infinite* Hurwitz matrix. It is known [1,5,4,2] that the total positivity of the matrix H_2 is equivalent to the positivity of the leading principal minors of H_2. In [3] (see also [6]), there were found finitely many sufficient conditions for the matrix H_M to be totally positive. In this talk, we show that positivity of finitely many certain minors of the generalized Hurwitz matrix H_M, $2 \leqslant M \leqslant n$, is necessary and sufficient for total positivity of the matrix H_M.

We also present some applications of totally positive generalized Hurwitz matrices to the root location of polynomials.

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Friday, 17:30 – 18:00

Speeding-up Convolutional Neural Networks Using Fine-tuned CP-Decomposition

Vadim Lebedev
Skotech

Convolutional neural networks are a dominating approach for many tasks in computer vision and image processing. However, multi-channel convolutions make them a very computationally intensive method. We propose an approach for speeding up convolutions within large convolutional neural networks based on tensor decomposition and discriminative fine-tuning. We use non-linear least squares to compute a low-rank CP-decomposition of the 4D convolution kernel tensor into a sum of a small number of rank-one tensors. At the second step, this decomposition is used to replace the original convolution with a sequence of four convolutions with smaller kernels. After such replacement, the entire network is fine-tuned on the training data using standard back-propagation process. We evaluate our approach and show that it is competitive with previous approaches, leading to higher obtained CPU speedups at the cost of lower accuracy drops.

On attainability of values for permanents of (0,1) matrices

Konstantin Taranin
Lomonosov Moscow State University

Let $\mathbb{R}$ and M_n denote the field of real numbers and $\mathbb{R}$-ring of $n \times n$ matrices respectively. $\Omega_n \in M_n$ is the set of $n \times n$ (0,1) matrices, that is, matrices whose entries consist only of integers 0 and 1. S_n stands for the group of substitutions on an n-set.

Definition 1. *Function* per : $M_n \to \mathbb{R}$ *such that for any* $A \in M_n$

$$\text{per}(A) = \sum_{\sigma \in S_n} a_{1\sigma(1)} \dots a_{n\sigma(n)},$$

where a_{ij} *are entries of* A*, is called permanent.*

It is easy to show that the maximal value of permanent for matrices from Ω_n is $n!$, and therefore all possible values of permanent, restricted on Ω_n, are some integers between 0 and $n!$. Not all of these numbers are permanents: for instance, nearest to $n!$ value of permanent (not equal to $n!$ itself) is $n! - (n-1)!$. Due to great difficulties in evaluating permanent it is reasonable to try to find out disposition of its values for Ω_n, given number n. It was proved in [1] that each integer from 0 to 2^{n-1} (inclusively) is a permanent of some matrix $A \in \Omega_n$. Thus, 2^{n-1} is a kind of upper bound for values of permanent that are consequent integers starting from zero. One of our results is that this bound is $\frac{11}{8}$ times greater than it is shown in [1]. Other results to be presented in the talk are the following.

Lemma 1. *The greatest odd value of permanent, calculated for matrices from* Ω_n*, is* $[\frac{(n+1)!}{ne}]$*, where* $[a] = \lfloor a + 0.5 \rfloor$ *for any* $a \in \mathbb{R}$*,* $\lfloor a \rfloor = N$ *for any* $N \le a < N+1$*,* $N \in \mathbb{N}$*, and* e *is a symbol for exponent.*

Lemma 2. *Given* $n > 1$*, the greatest not divisible by* 3 *value of permanent, calculated for matrices from* Ω_n*, is* $[\frac{(n-2)!}{e}(n^2+n-1)]$.

Theorem 1. *Given* $n > 2$*, the amount of values of permanent of matrices from* Ω_n *in interval* $(\frac{(n+1)!}{ne}, n!]$ *doesn't exceed*

$$\frac{e-2}{2}((n-1)! + (n-3)!) + \left[\frac{n}{2}\right] + (-1)^{n-1} \sum_{k=2}^{n} \frac{(-1)^k}{k}.$$

The author wants to thank his scientific advisor professor A.E.Guterman for formulating the problem, permanent attention to work upon it and valuable discussions. The work was financially supported by grants MD-962-2014.01 and RFBR 15-31-20329.

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Friday, 18:00 – 18:30

From low-rank approximation to an efficient rational Krylov subspace method for the Lyapunov equation

Denis Kolesnikov
Skolkovo Institute of Science and Technology

Many approaches for solving large scale algebraic Lyapunov equations with low-rank right-hand sides are based on rational Krylov subspaces. In this talk we propose a new method for the construction of a rational Krylov subspace and compare its efficiency to other approaches. We start from the connection between the approximation of the solution of the Lyapunov equation of the form $AX + XA^\top = -y_0 y_0^\top$, and the solution of a system of linear ODEs of the form $\frac{dy}{dt} = Ay, \quad y(0) = y_0$. Based on this we propose a new functional which minimization can be used to find low-rank approximation in non-symmetric case and compute its gradient. Using the gradient representation we propose a basis extension method that uses solution of an auxiliary Sylvester equation, and a rank-1 approximation to the solution of the Sylvester equation gives a very cheap and efficient method to compute the shifts for the rational Krylov subspace. We compare the new method (ALR) to two other approaches [1], [2] on several model problems and find that the new method has considerably fewer number of iterations (see Figure). The results will be presented in a forthcoming paper [3].

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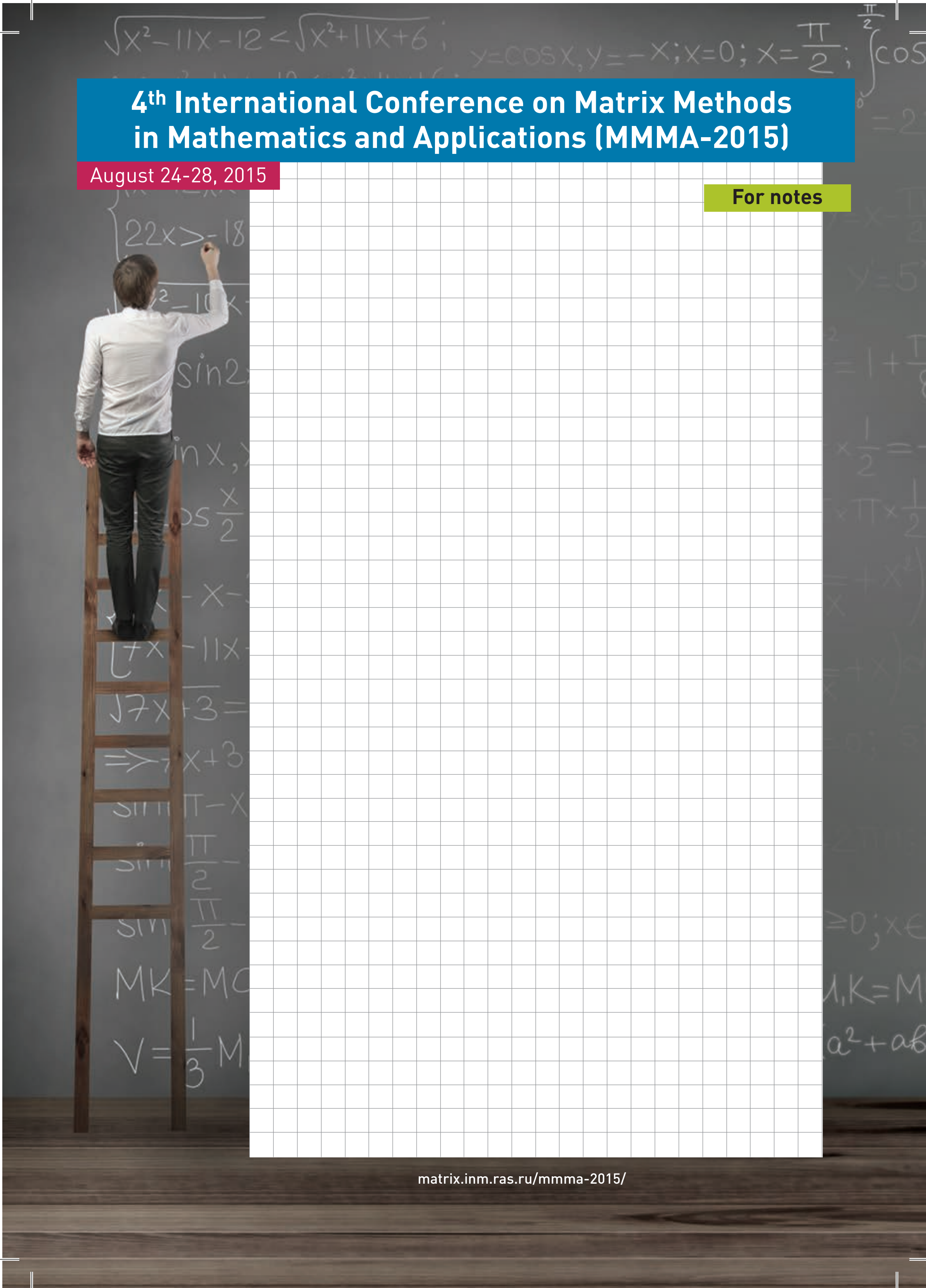
Friday, 18:00 – 18:30

Towards fast topological shape optimization with boundary elements

Igor Ostanin
Skolkovo Institute of Science and Technology

Wide class of engineering design tasks can be posed as optimization problems where the shape and topology of an elastic domain are optimized to reduce costs, e.g. global compliance, while satisfying certain constraints, such as volume constraint. We propose an application of a fast 3D boundary element code to the problems of shape and topology optimization. Our algorithm is based on the formalism of topological derivatives. Adaptive tree strategy of sampling of topological derivatives inside the domain, high performance algebraic solver and the analysis of optimization problem in reduced dimensions promise state of the art performance in the problems of engineering optimization. The approach can be applied to various optimization problems, such as minimization of compliance of an elastic structure or minimization of the distance from a current homogenized elasticity tensor of a periodic structure to the desired one. The efficiency of the approach is illustrated with a numerical example.



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