Steady State Solutions of Two-Sided Extremally-Linear Equation Systems.

Karel Zimmermann
Charles University Prague, Faculty of Mathematics and Physics

Tue 16:30–17:00
Room AV 01.12

We will consider equation system

$$\max_{j\in J}(a_{ij}+x_j) = \min_{k\in J}(b_{ik}+y_k),\ i\in I.\ j\in J, \tag{1}$$

where $I,\ J$ are finite index sets, $a_{ij}, b_{ik} \in R = (-\infty, +\infty)$ for all $i \in I,\ j,k \in J$. The system will be referred to as two-sided $(\max,+)/(\min,+)$-linear equation system. Solvability of the system will be studied and a method for finding so called steady state solution of the system, i.e. a solution with $y = x+\alpha,\ \alpha \in R$, will be proposed. Such solutions are called steady state solutrions. Steady state solutions for one-sided $(\max,+)-$ resp. $(\min,+)$-equation systems have been studied e.g. in the literature since the 60-ties of thye last century. The method proposed in this contribution uses for the construction of steady state solutions of the two-sided system $(\max,+)$- resp. $(\min,+)$- eigenvectors and eigenvalues of a special matrix Q, the elements of which depend on elements of matrices $A = (a_{ij}),\ B = (b_{ik})$. Especially it is proved that if $\lambda(Q)$ is the $(\min,+)$-eigenvalue of Q and $\tilde{x}$ the corresponding eigenvector, then $(\tilde{x}, \tilde{x}-\lambda(Q))$ is under an easily verifiable additional condition a steady state solution of (1). If the additional condition is not satisfied, no steady state solution of (1) exists. In the latter case only $\le$-inequality in (1) holds.

Application of the results to synchronization of cyclically repeated groups of activities with deterministic processing times is briefly discussed. The steady state solution gives a schedule, in which the difference between release times of all activities in two sequencial cycles is constant. Possibilities of future research especially the extension of the results to the case of bounded variables and solving two-sided systems with $(\max,+)$- linear equations on both sides of equations will be briefly mentioned.

Tropical optimization techniques for solving minimax location problems with Chebyshev and rectilinear distances

Nikolai Krivulin
St. Petersburg State University

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Room AV 01.12

We propose new algebraic solutions for constrained minimax single-facility location problems in multidimensional spaces with Chebyshev distance, and in the plane with rectilinear distance. We first formulate the location problems in a standard form, and outline existing solutions. Then, the problems are represented in terms of tropical (idempotent) algebra as optimization problems to minimize non-linear objective functions defined on vectors over an idempotent semifield, subject to linear vector inequality and equality constraints. We apply methods and techniques of tropical optimization to obtain direct, explicit solutions of the problems. The results obtained are used to derive solutions of the location problems under consideration in a closed form, which is ready for formal analysis and straightforward computation. We examine extensions of the approach to handle other problems, such as rectilinear single-facility location in high-dimensional spaces and multi-facility location. To illustrate, we present numerical solutions of example location problems and provide graphical representation of these solutions.