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An Analysis of Stochastic Project Scheduling Problems

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We examine a class of project scheduling problems with random processing times and limited continuous nonrenewable resources. It mainly includes stochastic problems concerning the allocation of such resources as money and energy. Also, certain manpower allocation problems may be referred to this class when the resources are represented in terms of man–hour per day or per week.

Consider a project which consists in processing n jobs according to some technological precedence constraints. For each job $i = 1, \ldots, n$, we define the processing time by a random variable $\tau_i(\theta, \omega)$, where $\theta \in \Theta$ is a real vector of available resources, and ω is a random vector which represents the random effect involved in realizing the project. Denote the completion time of job i by $t_i(\theta, \omega)$ and remark that it may be expressed in terms of the processing times $\tau_1, \ldots, \tau_n$ by means of operations $+$ and *max*. Moreover, the main functions, namely *makespan, weighted flowtime, tardiness cost* (see [2]), which one normally chooses as performance criteria of the project, also allows of representing them in the same form. We therefore restrict our class of stochastic scheduling problems to ones of minimizing the function $F(\theta) = E[f(\theta, \omega)]$, subject to $\theta \in \Theta$, where f is one of the above performance criteria, Θ is a convex closed subset of some Euclidian space.

Although f is described rather simply, one can hardly obtain the function F analytically in closed form. In that case, it is required to estimate F or its gradient so as to apply any optimization procedure. The next result provides conditions for an estimate of the gradient to be unbiased [1].

Theorem 1. Suppose that

(i) for each $\theta \in \Theta$, $\tau_i(\theta, \omega)$, $i = 1, \ldots, n$, are continuous and independent random variables;

(ii) for each $i = 1, \ldots, n$, $\tau_i(\theta, \omega)$ is a differentiable function of θ for almost all ω, and there exists a random variable $\lambda(\omega)$ with $E\lambda \leq \infty$, such that $|\tau_i(\theta_1, \omega) - \tau_i(\theta_2, \omega)| \leq \lambda(\omega)\|\theta_1 - \theta_2\|$ holds for all $\theta_1, \theta_2 \in \Theta$ and almost all ω.

Then $\nabla_\theta f(\theta, \omega)$ is an unbiased estimate of $\nabla F(\theta)$.

It is also useful the following theorem.

Theorem 2. If for each $i = 1, \ldots, n$, $\tau_i(\theta, \omega)$ is a convex function with respect to $\theta \in \Theta$ for almost all ω, then $F(\theta)$ is convex.

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The Simplest Game-Theoretical Model of Non-Balanced Market

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The dinamic model of non-balanced market with n agents is considered. A state of the system is described with m-tuples of commodities and the values of money for each agent. The strategy of each agent *at* a period t is defined from the results of period $(t-1)$ and has a random error. The agent also can receive a credite an each period, but when his debt is enough large, he "dies". The system is in stable state if the number of living agents becames stable. The optimal prices of a period t can be define as an equilibrium prices, or from Shapley-value, T-value and other solutions cousepts of corresponding cooperative market games.

In the paper initiale states and probability distributions for which the dynamic economic system is stable are investigated. The simplest interactive procedure is supplied.